

### *Laplace transform theorems*

1. Definition	$L[f(t)] = F(s) = \int_{0^-}^{\infty} f(t)e^{-st} dt$
2. Linearity	$L[k_1 f(t_1) + k_2 f(t_2)] = k_1 F_1(s) + k_2 F_2(s)$
3. Time shift	$L[f(t - \tau)] = e^{-s\tau} F(s)$
4. Frequency Shift	$L[e^{-at} f(t)] = F(s + a)$
5. Scaling Theorem	$L[f(at)] = \frac{1}{a} F\left(\frac{s}{a}\right)$
6. Differentiation Theorem	$L\left[\frac{df}{dt}\right] = sF(s) - f(0)$
7. Differentiation Theorem	$L\left[\frac{d^2 f}{dt^2}\right] = s^2 F(s) - sf(0) - \dot{f}(0)$
8. Differentiation Theorem	$L\left[\frac{d^n f}{dt^n}\right] = s^n F(s) - \sum_{k=1}^n s^{n-k} f^{(k-1)}(0)$
9. Integration Theorem	$L\left[\int f(\tau) d\tau\right] = \frac{F(s)}{s} + \frac{\int_{0^+} f(\tau) d\tau}{s}$
10. Final value theorem <sup>h</sup>	$f(\infty) = \lim_{s \rightarrow 0} sF(s)$
11. Initial value theorem <sup>3</sup>	$f(0^+) = \lim_{s \rightarrow \infty} sF(s)$
<p><sup>h</sup> Provided all poles of <math>F(s)</math> have negative real parts with the exception of possibly one pole at the origin.</p> <p><sup>3</sup> Provided <math>f(t)</math> is continuous or has a step discontinuity at <math>t = 0</math>.</p>	

**Laplace transform of time funtions**

1. $\mathbf{d}(t)$	1
2. $u(t)$	$1/s$
3. $tu(t)$	$1/s^2$
4. $\frac{1}{2!}t^2u(t)$	$1/s^3$
5. $\frac{1}{(m-1)!}t^{m-1}u(t)$	$1/(s^m)$
6. $e^{-at}u(t)$	$1/(s+a)$
7. $te^{-at}u(t)$	$1/(s+a)^2$
8. $\frac{1}{(m-1)!}t^{m-1}e^{-at}u(t)$	$1/(s+a)^m$
9. $(1-e^{-at})u(t)$	$a/[s(s+a)]$
10. $\frac{1}{a}(at-1+e^{-at})u(t)$	$a/[s^2(s+a)]$
11. $(1-at)e^{-at}u(t)$	$s/(s+a)^2$
12. $\sin(\mathbf{w}t)u(t)$	$\mathbf{w}/(s^2+\mathbf{w}^2)$
13. $\cos(\mathbf{w}t)u(t)$	$s/(s^2+\mathbf{w}^2)$
14. $e^{-at}\cos(\mathbf{w}t)u(t)$	$(s+a)/[(s+a)^2+\mathbf{w}^2]$
15. $e^{-at}\sin(\mathbf{w}t)u(t)$	$\mathbf{w}/[(s+a)^2+\mathbf{w}^2]$
16. $\left\{1 - \frac{1}{\sqrt{1-z^2}} e^{-z\mathbf{w}_n t} [\sin(\mathbf{w}_d t + \mathbf{q})]\right\}u(t)$	
$\mathbf{w}_d = \mathbf{w}_n \sqrt{1-z^2}$ ; $\mathbf{q} = \cos^{-1}(z)$	
OR $\left\{1 - e^{-z\mathbf{w}_n t} \left[ \cos(\mathbf{w}_d t) + \frac{z}{\sqrt{1-z^2}} \sin(\mathbf{w}_d t) \right]\right\}$	$\frac{\mathbf{w}_n^2}{s(s^2 + 2z\mathbf{w}_n s + \mathbf{w}_n^2)}$