Table 3-4 Fourier Transforms of Energy Signals

	$x(t)$	$X(\omega)$	$\lvert X(\omega)\rvert$
Rect(t)	$u\left(t+\frac{1}{2}\right) - u\left(t-\frac{1}{2}\right)$	$\frac{\sin(\omega/2)}{\omega/2}$	
Exponential	$e^{-at}u(t)$	$\frac{1}{j\omega + a}$	
Triangular	$1-\lvert t\rvert, \lvert t\rvert < 1$; 0 elsewhere	$\left[\frac{\sin \omega/2}{\omega/2}\right]^2$	
Gaussian	$e^{-a^2t^2}$	$\frac{\sqrt{\pi}}{a} e^{-(\omega^2/4a^2)}$	
Double exponential	$e^{-a\lvert t\rvert}$	$\frac{2a}{a^2+\omega^2}$	
Damped sine	$e^{-at}\sin(\omega_0 t)u(t)$	$\frac{\omega_0}{(a+j\omega)^2+\omega_0^2}$	
Damped cosine	$e^{-at}\cos(\omega_0 t)u(t)$	$\frac{a+j\omega}{(a+j\omega)^2+\omega_0^2}$	
—	$\frac{1}{\beta-\alpha}[e^{-\alpha t} - e^{-\beta t}]u(t)$	$\frac{1}{(j\omega+\alpha)(j\omega+\beta)}$	
Cosine pulse	$\cos \omega_0 t\left[u\left(t+\frac{T}{2}\right) - u\left(t-\frac{T}{2}\right)\right]$	$\frac{T}{2}\left[\frac{\sin(\omega-\omega_0)T/2}{(\omega-\omega_0)T/2} + \frac{\sin(\omega+\omega_0)T/2}{(\omega+\omega_0)T/2}\right]$	

Table 3-5 Fourier Transforms of Power Signals

	$x(t)$	$X(\omega)$	$\lvert X(\omega)\rvert$
Unit impulse	$\delta(t)$	1	
Unit doublet	$\delta'(t)$	$j\omega$	
Unit step	$u(t)$	$\pi\delta(\omega) + \frac{1}{j\omega}$	
Signum function	$\operatorname{sgn} t = \frac{t}{\lvert t\rvert}$	$\frac{2}{j\omega}$	
Constant	K	$2\pi K\delta(\omega)$	
Cosine wave	$\cos \omega_0 t$	$\pi[\delta(\omega+\omega_0) + \delta(\omega-\omega_0)]$	
Sine wave	$\sin \omega_0 t$	$j\pi[\delta(\omega+\omega_0) - \delta(\omega-\omega_0)]$	
Periodic wave	$\sum_n x_T(t-nT)$	$\frac{2\pi}{T}\sum_n X_T\left(\frac{2\pi n}{T}\right)\delta\left(\omega-\frac{2\pi n}{T}\right)$	
Impulse train	$\Sigma\delta(t-nT)$	$\frac{2\pi}{T}\Sigma\delta\left(\omega-\frac{2\pi n}{T}\right)$	
Complex sinusoid	$e^{j\omega_0 t}$	$2\pi\delta(\omega-\omega_0)$	
Unit ramp	$tu(t)$	$j\pi\delta'(\omega) - \frac{1}{\omega^2}$	

Table 3-3 Fourier Transforms of Mathematical Operations

Operation	$x(t)$	$X(\omega)$	$X(f)$
Transformation	$x(t)$	$\int_{-\infty}^{\infty} x(t)e^{-j\omega t}\,dt$	$\int_{-\infty}^{\infty} x(t)e^{-j2\pi ft}\,dt$
Inversion	$\frac{1}{2\pi}\int_{-\infty}^{\infty} X(\omega)e^{j\omega t}\,d\omega$	$X(\omega)$	$X(f)$
Superposition	$a_1x_1(t) + a_2x_2(t)$	$a_1X_1(\omega) + a_2X_2(\omega)$	$a_1X_1(f) + a_2X_2(f)$
Reversal	$x(-t)$	$X(-\omega)$	$X(-f)$
Symmetry	$X(t)$	$2\pi x(-\omega)$	$x(-f)$
Scaling	$x(at)$	$\frac{1}{\lvert a\rvert} X\left(\frac{\omega}{a}\right)$	$\frac{1}{\lvert a\rvert} X\left(\frac{f}{a}\right)$
Delay	$x(t - t_0)$	$e^{-j\omega t_0}X(\omega)$	$e^{-j2\pi ft_0}X(f)$
Modulation	$e^{j\omega_0 t}x(t)$	$X(\omega - \omega_0)$	$X(f - f_0)$
Time differentiation	$\frac{d^n}{dt^n}x(t)$	$(j\omega)^nX(\omega)$	$(j2\pi f)^nX(f)$
Frequency differentiation	$t^nx(t)$	$(j)^n\frac{d^n}{d\omega^n}X(\omega)$	$\left(\frac{j}{2\pi}\right)^n\frac{d^n}{df^n}X(f)$
Integration	$\int_0^t x_1(t)\,dt + \int_{-\infty}^{t} x_0(t)\,dt$	$\frac{1}{j\omega}X(\omega)$	$\frac{1}{j2\pi f}X(f)$
Integration	$\int_{-\infty}^{t} x(t)\,dt$	$\frac{1}{j\omega}X(\omega) + \pi X(0)\delta(\omega)$	$\frac{1}{j2\pi f}X(f) + \frac{1}{2}X(0)\delta(f)$
Convolution	$x_1 * x_2 = \int_{-\infty}^{\infty} x_1(\lambda)x_2(t-\lambda)\,d\lambda$	$X_1(\omega)X_2(\omega)$	$X_1(f)X_2(f)$
Multiplication	$x_1(t)x_2(t)$	$\frac{1}{2\pi}\int_{-\infty}^{\infty} X_1(\xi)X_2(\omega - \xi)\,d\xi$	$\int_{-\infty}^{\infty} X_1(\xi)X_2(f - \xi)\,d\xi$

This impulse response is shown in Fig. 3-37b. It is clear from the figure that this is not a physically realizable system, because the output occurs prior to application of the input. One way of approximating the ideal filter response is to employ a system having a response similar in form to that of Fig. 3-37b but delayed in time. The greater the delay, the more nearly the shape of Fig. 3-37b can be reproduced by a physically realizable filter. The effect of the delay in the frequency domain is to produce a phase shift that varies linearly with frequency.

Convolution in the frequency domain. It is shown in Sec. 3-10 that convolution in the time domain corresponds to multiplication in the frequency domain. From the symmetry properties of the Fourier transform, it follows that convolution

of two transforms in the frequency domain correspon
original functions in the time domain. The exact relatio

$$x_1(t)x_2(t) \Leftrightarrow \frac{1}{2\pi}\int_{-\infty}^{\infty} X_1(\xi)X_2(\omega - \xi)\,d\xi = \frac{1}{2\pi}$$

$$x_1(t)x_2(t) \Leftrightarrow X_1(f) * X_2(f)$$

These expressions can be verified formally by invertin
relationships given in (3-146) and (3-147) are particul
modulation processes when one of the time functions
of impulses. Examples of this will be considered in lat

A collection of the most frequently used Fourier
given in Table 3-3. Table 3-4 lists a number of eleme
ditional transform pairs are given in Table 3-5 that rel
average power. These and other useful tables are coll
A. More extensive tabulations of Fourier transforms are
listed at the end of the chapter.

3-14 FOURIER TRANSFORMS OF POWER SIGNALS

The ordinary Fourier transform is limited to the trans
are absolutely integrable—that is, functions that obey

$$\int_{-\infty}^{\infty} |x(t)|\,dt < \infty$$

A number of functions having great usefulness do no
example, a sine wave or a step function does not.
nevertheless, be handled by allowing the Fourier trans
in some cases, higher-order singularity functions. Thi
rigorous mathematical basis by means of the theory
however, it will be sufficient for our purposes to just
ering the impulse as a limiting form of a proper fu
correct results are obtained when this method is used.

Consider the function sgn(t), called signum t, wh

$$\text{sgn}(t) = \begin{cases} -1 & t < 0 \\ 0 & t = 0 \\ +1 & t > 0 \end{cases}$$

[1] A. H. Zemanian, *Distribution Theory and Transform Analysis*, N
pany, Inc., 1965.