1. $$\frac{d^2x}{dt^2} + 10\frac{dx}{dt} + 21x = 8u(t)$$

$$\frac{d^2x}{dt^2} \xleftrightarrow{\mathcal{L}} s^2X(s) - s\,x(0) - \dot{x}(0)$$

$$\frac{dx}{dt} \xleftrightarrow{\mathcal{L}} sX(s) - x(0)$$

$$x \xleftrightarrow{\mathcal{L}} X(s)$$

$$u(t) \xleftrightarrow{\mathcal{L}} 1/s$$

rewrite:

$$s^2X(s) - s\,x(0) - \dot{x}(0) + 10[sX(s) - x(0)] + 21X(s) = 8/s$$

$$X(s)[s^2 + 10s + 21] - s\,x(0) - \dot{x}(0) - 10x(0) = 8/s$$

assume I.C.'s are 0

$$X(s) = \frac{8}{s[s^2+10s+21]}$$

$$= \frac{8}{s(s+7)(s+3)}$$

$$= \frac{K_1}{s} + \frac{K_2}{s+7} + \frac{K_3}{s+3}$$

$$K_1 = \lim_{s \to 0} \frac{8}{(s+7)(s+3)} = 8/21$$

$$K_2 = \lim_{s \to -7} \frac{8}{s(s+3)} = 8/28$$

$$K_3 = \lim_{s \to -3} \frac{8}{s(s+7)} = -8/12$$

$$X(s) = \frac{\frac{8}{21}}{s} + \frac{\frac{8}{28}}{s+7} + \frac{-\frac{8}{12}}{s+3}$$

$$x(t) = \left[\frac{8}{21} + \frac{8}{28}e^{-7t} - \frac{8}{12}e^{-3t}\right]u(t)$$

verify

$$\frac{dx}{dt} = \left(\frac{8}{21} + \frac{8}{28}e^{-7t} - \frac{8}{12}e^{-3t}\right)\delta(t) + \left(-2e^{-7t} + 2e^{-3t}\right)u(t)$$

$$\frac{d^2x}{dt^2} = \left(\frac{8}{21} + \frac{8}{28}e^{-7t} - \frac{8}{12}e^{-3t}\right)\delta'(t) + \left(-2e^{-7t} + 2e^{-3t}\right)\delta(t) + \left(-2e^{-7t} + 2e^{-3t}\right)\delta(t) + \left(14e^{-7t} - 6e^{-3t}\right)u(t)$$

$$\frac{d^2x}{dt^2} + 10\frac{dx}{dt} + 21\,x(t) =$$

$$\delta'(t)\left[\frac{8}{21} + \frac{8}{28}e^{-7t} - \frac{8}{12}e^{-3t}\right] +$$

$$\delta(t)\left[10\left(\frac{8}{21} + \frac{8}{28}e^{-7t} - \frac{8}{12}e^{-3t}\right) + 2\left(-2e^{-7t} + 2e^{-3t}\right)\right] +$$

$$u(t)\Big[\left(-2e^{-7t} + 2e^{-3t}\right)10 + 14e^{-7t} - 6e^{-3t} + 21\left(\frac{8}{21} + \frac{8}{28}e^{-7t} - \frac{8}{12}e^{-3t}\right)\Big]$$

$$= \delta(t)\left[\text{stuff}\right] +$$

$$\delta'(t)\left[\text{stuff}\right] +$$

$$u(t)\left[-20e^{-7t} + 20e^{-3t} + 14e^{-7t} - 6e^{-3t} + 8 + \frac{168}{28}e^{-7t} - \frac{168}{12}e^{-3t}\right]$$

$$= \delta(t)\left[\text{stuff}\right] + \delta'(t)\left[\text{stuff}\right] + u(t)\left[e^{-7t}\left(-20 + 14 + 6\right) + e^{-3t}\left(20 - 6 - 14\right)e^{-3t} + 8\right]$$

$$= \delta(t)\left[\text{stuff}\right] + \delta'(t)\left[\text{stuff}\right] + 8\,u(t)$$

at $t=0$

$$10\left(\frac{8}{21} + \frac{8}{28} - \frac{8}{12} - 4 + 4\right) = 0$$

at $t=0$

$$\left(\frac{8}{21} + \frac{8}{28} - \frac{8}{12}\right) = 0$$

$= 8\,u(t)$

Since $\delta(t) = 0 \quad \forall\, t \neq 0$
and when $t=0$ the coefficient $=0$

$\delta'(t) = 0 \quad \forall\, t \neq 0$
and when $t=0$ the coefficient $=0$

2a. $$\frac{2s}{(s+3)(s+7)} = \frac{-6/4}{s+3} + \frac{+14/4}{s+7}$$

$$\Big\downarrow \mathcal{L}^{-1}$$

$$\left(-1.5\, e^{-3t} + 3.5\, e^{-7t}\right) u(t)$$

b. $$\frac{2s}{(s+3)(s+7)(s+10)} = \frac{-6/28}{s+3} + \frac{-14/-12}{s+7} + \frac{-20/21}{s+10}$$

$$\Big\downarrow \mathcal{L}^{-1}$$

$$\left(-\tfrac{3}{14}\, e^{-3t} + \tfrac{7}{6}\, e^{-7t} - \tfrac{20}{21}\, e^{-10t}\right) u(t)$$

c. $$\frac{2s}{s^2+10s+50} = \frac{2s}{(s+5)^2+5^2}$$

$$= 2\,\frac{(s+5)}{(s+5)^2+5^2} + \left(\frac{-10}{5}\right)\frac{5}{(s+5)^2+5^2}$$

$$\Big\downarrow \mathcal{L}^{-1}$$

$$= \left(2\, e^{-5t} \cos 5t - 2\, e^{-5t} \sin 5t\right) u(t)$$

d. $$\frac{s+7}{s^2+s+1} = \frac{s+7}{(s+1/2)^2+\left(\sqrt{3/4}\right)^2}$$

$$= (1)\,\frac{s+1/2}{(s+1/2)^2+\left(\sqrt{3/4}\right)^2} + \left(\frac{6.5}{\sqrt{3/4}}\right)\frac{\sqrt{3/4}}{(s+1/2)^2+\left(\sqrt{3/4}\right)^2}$$

$$\Big\downarrow \mathcal{L}^{-1}$$

$$\left[e^{-1/2\,t} \cos \sqrt{\tfrac{3}{4}}\,t + \frac{6.5}{\sqrt{3/4}}\, e^{-1/2\,t} \sin \sqrt{\tfrac{3}{4}}\,t\right] u(t)$$

3. $$H(s) = \frac{1}{s^2+3s+7}$$

$$Y(s) = X(s)\ H(s)$$

$$= \frac{1}{s(s^2+3s+7)}$$

$$= \frac{1/7}{s} + \frac{-\frac{1}{7}s - \frac{3}{7}}{s^2+3s+7}$$

$$= \frac{1/7}{s} + \frac{C_1(s+1.5) + C_2(\sqrt{4.75})}{(s+1.5)^2 + \sqrt{4.75}^2}$$

$$C_1 = -1/7$$

$$C_2 = \frac{-3/7 + \frac{1.5}{7}}{\sqrt{4.75}} = -.0983$$

$$= \frac{1/7}{s} + \left(-\frac{1}{7}\right)\frac{s+1.5}{(s+1.5)^2+\sqrt{4.75}^2} + (-.0983)\frac{\sqrt{4.75}}{(s+1.5)^2+\sqrt{4.75}^2}$$

$$\downarrow \mathcal{L}^{-1}$$

$$\left[\frac{1}{7} - \frac{1}{7}e^{-1.5t}\cos\sqrt{4.75}\,t - .0983\,e^{-1.5t}\sin\sqrt{4.75}\,t\right]u(t)$$