1a) $x(2t+1) \longleftrightarrow\ ?$

we know $x(at) \longleftrightarrow \frac{1}{|a|} X\left(\frac{\omega}{a}\right)$

and $x(t-t_0) \longleftrightarrow e^{-j\omega t_0} X(\omega)$

$\therefore\ x(at-t_0) \longleftrightarrow \dfrac{e^{-j(\omega/|a|)t_0} X\left(\frac{\omega}{a}\right)}{|a|}$

in this case $a=2$ & $t_0=-1$

$$\frac{1}{2} \cdot e^{-j(\omega/2)(-1)} \cdot \frac{\sin(\omega/4)}{\omega/4}$$

$$= \frac{2e^{+j\omega/2}\sin(\omega/4)}{\omega}$$

1b) $e^{-j2t}x(t+1) \longleftrightarrow\ ?$

we know $e^{+j\omega_0 t}x(t) \longleftrightarrow X(\omega-\omega_0)$

and $x(t-t_0) \longleftrightarrow X(\omega)e^{-j\omega t_0}$

$\therefore\ e^{-j\omega_0 t}x(t-t_0) \longleftrightarrow e^{-j(\omega-\omega_0)t_0} X(\omega-\omega_0)$

here $t_0=-1$ $\omega_0=-2$, so

$$e^{j(\omega+2)} X(\omega+2)$$

$$\frac{e^{j(\omega+2)}\sin\left(\frac{\omega+2}{2}\right)}{(\omega+2/2)}$$

c) $\frac{dx}{dt} \longleftrightarrow (j\omega) X(\omega)$

$$= \frac{j\omega \sin(\omega/2)}{\omega/2} = \underline{2j\sin(\omega/2)}$$

d) $x(-2t) \longleftrightarrow \frac{1}{2} X\left(\frac{\omega}{-2}\right)$

$$= \frac{\frac{1}{2}\sin\left(\frac{\omega/-2}{2}\right)}{(\omega/-2)/2} = \frac{2\sin(\omega/4)}{\omega}$$

e) $x(1-t) = x(-t+1)$

$$x(at - t_0) \longleftrightarrow \frac{1}{|a|} X\left(\frac{\omega}{a}\right) e^{-jt_0(\omega/a)}$$

$$\longleftrightarrow \frac{1}{1} \frac{\sin(-\omega/2)}{-\omega/2} e^{-j(-\omega)(1)}$$

$$= \frac{\sin(\omega/2)}{\omega/2} e^{j\omega}$$

f) $x(t)\cos(t) = x(t)\left[\frac{e^{jt}+e^{-jt}}{2}\right]$

$$= \frac{1}{2}e^{jt}x(t) + \frac{1}{2}e^{-jt}x(t)$$

$$\longleftrightarrow \frac{1}{2}X(\omega-1) + \frac{1}{2}X(\omega+1)$$

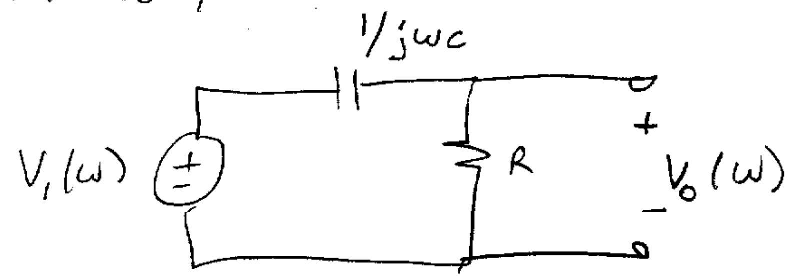
$$= \frac{1}{2}\left[\frac{\sin\left(\frac{\omega-1}{2}\right)}{\frac{\omega-1}{2}} + \frac{\sin\left(\frac{\omega+1}{2}\right)}{\frac{\omega+1}{2}}\right]$$

$$= \left[\frac{\sin\left(\frac{\omega-1}{2}\right)}{\omega-1} + \frac{\sin\left(\frac{\omega+1}{2}\right)}{\omega+1}\right]$$

2. From the tables,

$$x(t) = u(t + 1/2) - u(t - 1/2)$$
$$= \text{rect}(t)$$

3. Quick way:



$$\frac{V_o(\omega)}{V_1(\omega)} = \frac{R}{R + 1/jwc} = \frac{Rjwc}{Rjwc + 1}$$

Longer way:

$$-V_1(t) + V_c(t) + V_R(t) = 0$$
$$-V_1(t) + V_c(t) + iR = 0$$
$$V_i(t) = V_c(t) + C\frac{dV_c(t)}{dt}R$$
$$\downarrow \mathcal{F}$$
$$V_1(\omega) = V_c(\omega) + CR(j\omega)V_c(\omega)$$
$$V_c(\omega) = \frac{V_1(\omega)}{j\omega RC + 1}$$

3 Cont,

In this case, $V_O = V_R$, so

$$V_O = V_I - V_C$$

$$V_O(\omega) = V_I(\omega) - \frac{V_I(\omega)}{1+j\omega RC}$$

$$= \frac{(1+j\omega RC)V_I(\omega) - V_I(\omega)}{1+j\omega RC}$$

$$= \frac{j\omega RC}{1+j\omega RC} V_I(\omega)$$

and $$\frac{V_O(\omega)}{V_I(\omega)} = \frac{j\omega RC}{1+j\omega RC}$$

The impulse response happens when $V_I(\omega) = 1$, or

$$V_O(\omega) = \frac{j\omega RC}{1+j\omega RC} = RC\left[\frac{j\omega}{1+j\omega RC}\right]$$

$$= \frac{j\omega}{\frac{1}{RC} + j\omega}$$

3 Cont.

We know

$$e^{-at}u(t) \longleftrightarrow \frac{1}{j\omega + \alpha}$$

by the differentiation property,

$$\frac{d}{dt} e^{-at} u(t) \longleftrightarrow \frac{j\omega}{j\omega + \alpha}$$

so $$V_o(\omega) = \left(\frac{j\omega}{\frac{1}{RC} + j\omega} \right)$$

$$\text{hws } V_o(t) = \frac{d}{dt}\left(e^{-\frac{1}{RC}t} u(t) \right)$$

$$= e^{-\frac{1}{RC}t}\delta(t) + u(t)\left(\frac{-1}{RC}\right) e^{-\frac{1}{RC}t}$$

$$= e^{-\frac{1}{RC}t}\left\{ \delta(t) - \frac{1}{RC} u(t) \right\}$$

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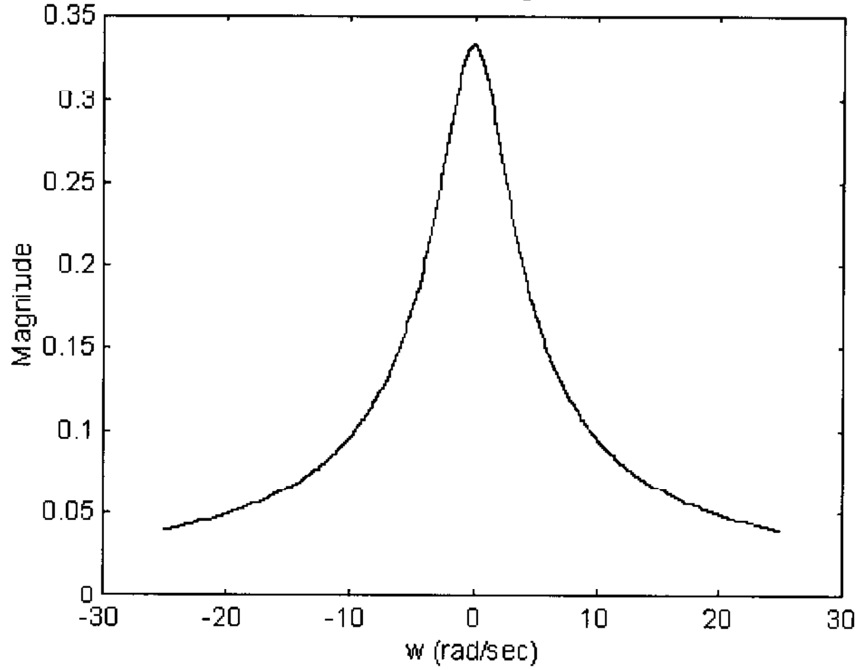
Problem 4 - Magnitude



Problem 4 - Phase