Thoughts on Homework 1, problem 2c

1 – We decided that it made sense to break up the problem into a set of nonoverlapping intervals

t<=-2	f1*f2=0
-2<t<=-1	Some Overlap
-1<t<=0	Some Overlap
0<t<=1	Some Overlap
1<t<=2	Some Overlap
t>2	f1*f2=0

2 – On the interval –2<t<=-1, the overlap is between the rising edge of f1(λ) = (2λ+2) and the falling edge of f2(t-λ) = (2t+2-2λ). We perform the integration there:

$$= \int_{-1}^{t+1} (2l + 2)(2t + 2 - 2l)\,dl$$

$$= 4\int_{-1}^{t+1} (l + 1)(t + 1 - l)\,dl$$

$$= 4\int_{-1}^{t+1} (-l^2 + tl + t + 1)\,dl$$

$$= 4\left[\frac{-l^3}{3} + t\frac{l^2}{2} + (t+1)l\right]_{-1}^{t+1}$$

$$= 4\left[\frac{-(t+1)^3 - -(-1)^3}{3} + \frac{t\left[(t+1)^2 - 1\right]}{2} + (t+1)\left[(t+1) - (-1)\right]\right]$$

$$= 4\left[\frac{-(t+1)^3 - 1}{3} + \frac{t\left[(t+1)^2 - 1\right]}{2} + (t+1)(t+2)\right]$$

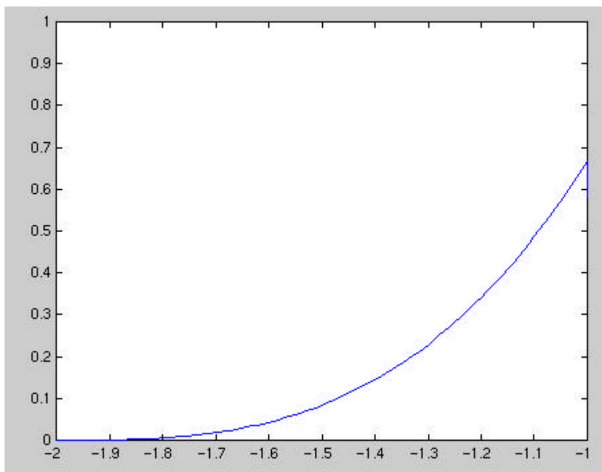
$$= 4\left[\frac{-(t^3 + 3t^2 + 3t + 1) - 1}{3} + \frac{t(t^2 + 2t + 1 - 1)}{2} + t^2 + 3t + 2\right]$$

$$= 4\left[\frac{-t^3 - 3t^2 - 3t - 2}{3} + \frac{t^3 + 2t^2}{2} + t^2 + 3t + 2\right]$$

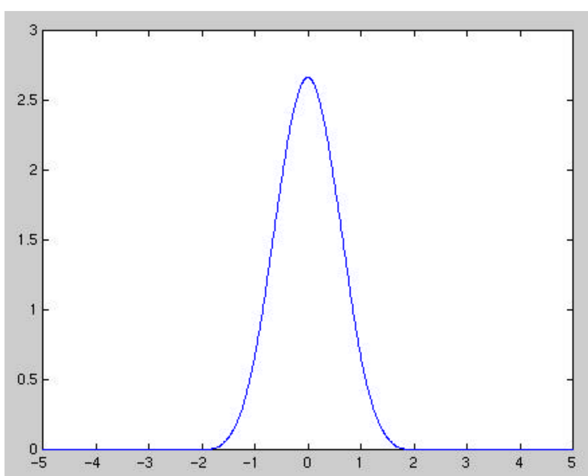
$$= \frac{4}{6}\left[-2t^3 - 6t^2 - 6t - 4 + 3t^3 + 6t^2 + 6t^2 + 18t + 12\right]$$

$$= \frac{4}{6}\left[t^3 + 6t^2 + 12t + 8\right]$$

This gives us the convolution of f1 and f2 *on the interval t=-2…-1 only*. Here it is displayed graphically:



The complete results of convolving the two functions is shown below:



For those of you that are interested, here is the MatLab code that I used to ***verify*** my answer (this might be something you'd be interested in doing with a take home exam).

```
% First, set up the triangle functions:
counter=0;
for t=-2.5:.01:2.5
  counter=counter+1;
  if (t<-1)
        f(counter)=0;
  elseif (t<0)
        f(counter)=2+2*t;
  elseif (t<1)
        f(counter)=2-2*t;
  else
        f(counter)=0;
  end
end

% Convolve the two functions:
% when using conv in matlab, you will get a vector back that is
% twice as long as what you had when you started, so we need to
% adjust our time axis. Furthermore, the answer from conv will be
% scaled by a factor 1/sampling rate - in this case 1/.01 = 100
t2=[-5:.01:5];
a=conv(f,f);
plot(t2,a/100);
hold on

% now let's verify the equation I generated mathematically matches
% MatLab's answers:
t3=[-2:.01:-1];
b=(2/3)*(t3.^3+6.*t3.^2+12.*t3+8);
plot(t3,b,'g');

% It does!
```