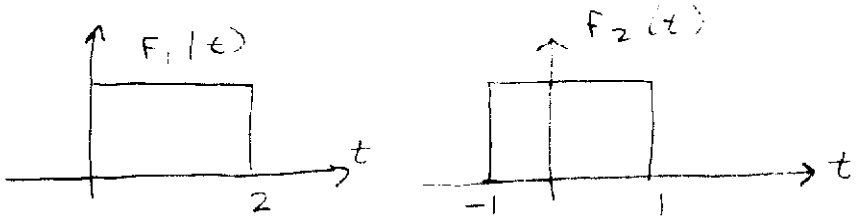
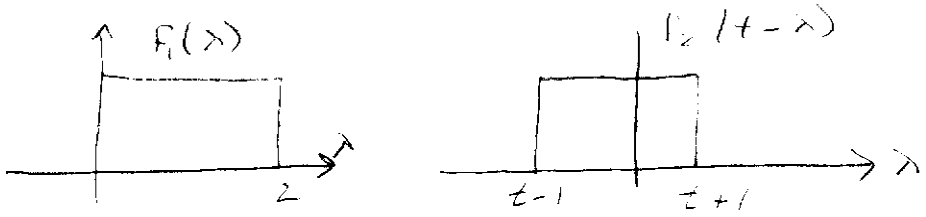
2a.

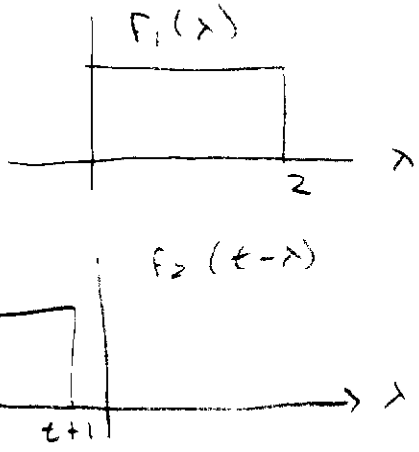


First, redraw as $f_1(\lambda)$ and $f_2(t-\lambda)$



Next, break this into 4 separate problems.

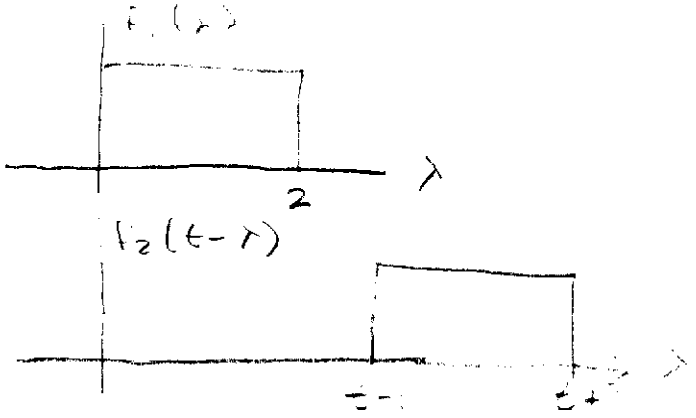
1- $t+1 < 0$ (or $t < -1$)



No overlap, so $f_1 * f_2 = 0$

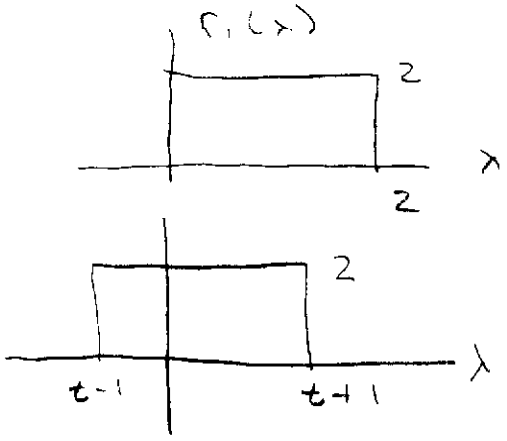
2a Cont.

2- $t-1 > 2$ (or $t > 3$)



No overlap, so $f_1 * f_2 = 0$

3- $(t+1)$ between 0 and 2 $(-1 < t < 1)$

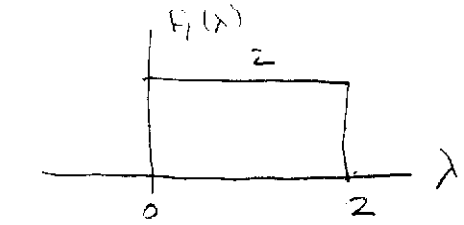


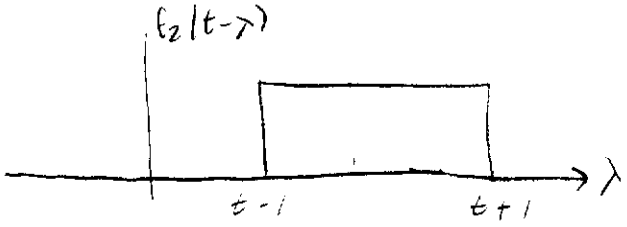
Some overlap - between 0 and $t+1$

$$f_1 * f_2 = \int_0^{t+1} 1 \, d\lambda = 4\lambda \Big|_0^{t+1} = 4(t+1)$$

2a Cont.

4- (t-1) between 0 and 2 ($1 < t < 3$)





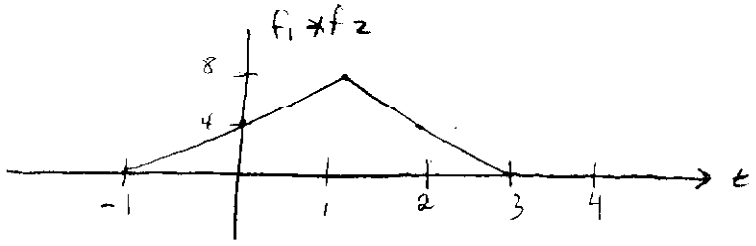
some overlap:

$$f_1 * f_2 = \int_{t-1}^{2} 4\, d\lambda = 4\lambda \Big|_{t-1}^{2} =$$

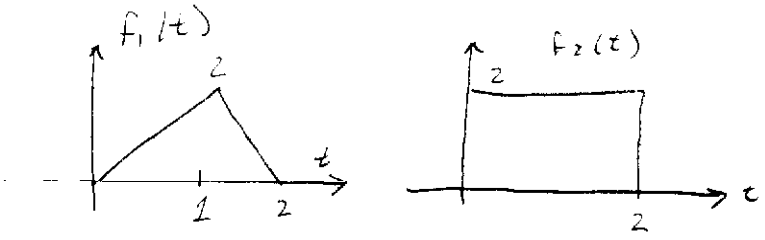
$$4\left[2 - t + 1\right] = 4\left[3 - t\right]$$

ans $f_1 * f_2 =$

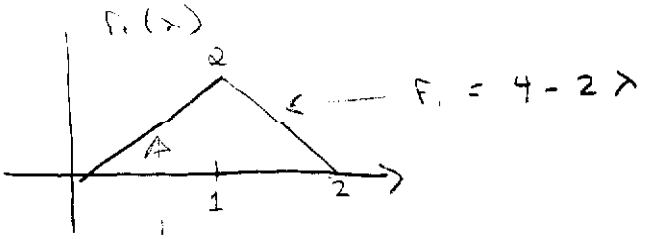
$$\begin{cases} 0 & t \le -1 \\ 4(t+1) & -1 < t \le 1 \\ 4(3-t) & 1 < t \le 3 \\ 0 & t > 3 \end{cases}$$



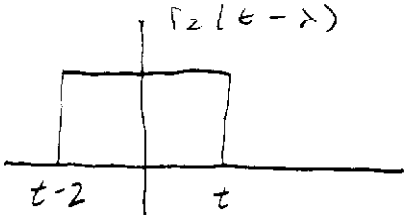
2b



first, write $f_1(\tau)$ & $f_2(t-\tau)$



$f_1 = 2\tau$



Now break up the problem into sub-problems

Clearly $t \leq 0 \Rightarrow f_1 * f_2 = 0$

$t - 2 > 2 \Rightarrow f_1 * f_2 = 0$

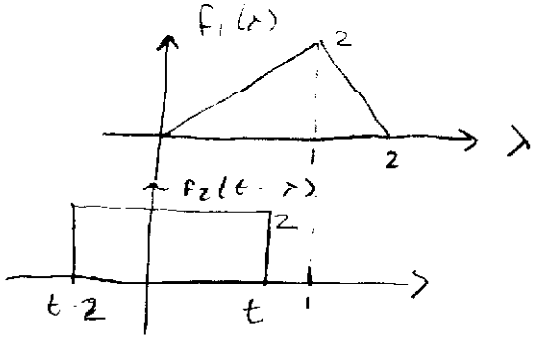
$(t > 4)$

Now consider separately the regions

1. $t = 0 \ldots 1$
2. $t = 1 \ldots 2$
3. $t = 2 \ldots 3$
4. $t = 3 \ldots 4$

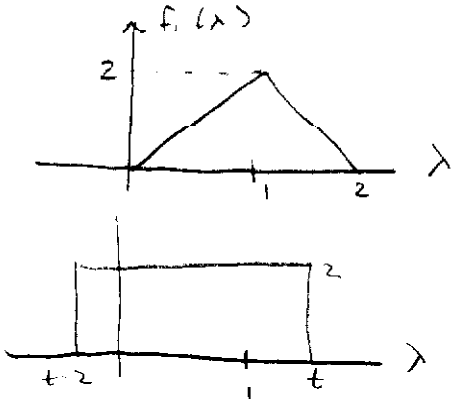
2b Cont

1- $0 < t \leq 1$



$$f_1 * f_2 = \int_0^t 2 \cdot (2\lambda)\, d\lambda = 2\lambda^2 \Big|_0^t = 2t^2$$

2- $1 < t \leq 2$



$$f_1 * f_2 = \int_0^1 2 \cdot 2\lambda\, d\lambda + \int_1^t 2 \cdot (4-2\lambda)\, d\lambda$$

$$= 2 + \int_1^t (8-4\lambda)\, d\lambda$$

2b Cont

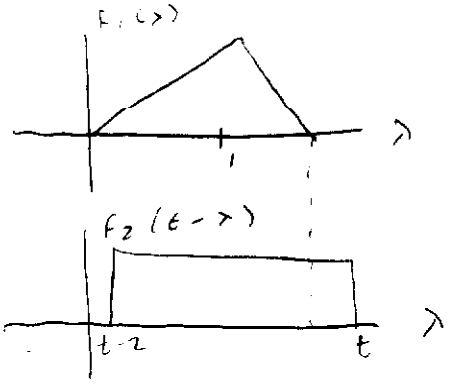
$$= 2 + (8\lambda - 2\lambda^2)\Big|_1^t$$

$$= 2 + (8t - 2t^2) - (8 - 2)$$

$$= 2 + 8t - 2t^2 - 6$$

$$= \underline{-2t^2 + 8t - 4}$$

3 - $2 < t \leq 3$



$$f_1 * f_2 = \int_{t-2}^{1} 2 \cdot 2\lambda \, d\lambda + \int_{1}^{2} 2 \cdot (4 - 2\lambda) \, d\lambda$$

$$= \int_{t-2}^{1} 4\lambda \, d\lambda + 2$$

$$= 2\lambda^2 \Big|_{t-2}^{1} + 2$$

$$= 2 - 2(t-2)^2 + 2$$

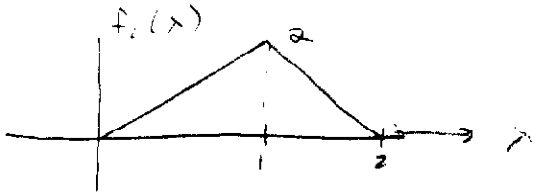
2b Cont

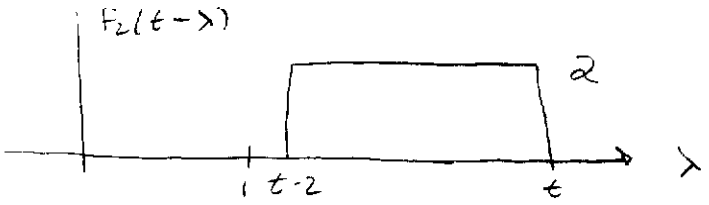
$$= 4 - 2(t^2 - 4t + 4)$$

$$= 4 - 2t^2 + 8t - 8$$

$$= \underline{-2t^2 + 8t - 4}$$

4 - $3 < t \leq 4$





$$f_1 * f_2 = \int_{t-2}^{2} 2(4 - 2\lambda)\, d\lambda$$

$$= \int_{t-2}^{2} (8 - 4\lambda)\, d\lambda = 8\lambda - 2\lambda^2 \Big|_{t-2}^{2}$$

$$= (16 - 8) - (8(t-2) - 2(t-2)^2)$$

$$= 8 - [8t - 16 - 2(t^2 - 4t + 4)]$$

$$= 8 - [8t - 16 - 2t^2 + 8t - 8)]$$

2b cont

$$= 8 - 8t + 16 + 2t^2 - 8t + 8$$

$$= 32 - 16t + 2t^2$$

$$= \underline{2t^2 - 16t + 32}$$

So $f_1 * f_2 =$

$$\begin{cases} 0 & t \le 0 \\ 2t^2 & 0 < t \le 1 \\ -2t^2 + 8t - 4 & 1 < t \le 3 \\ 2t^2 - 16t + 32 & 3 < t \le 4 \\ 0 & t > 4 \end{cases}$$