KEY - FINAL Exam

1. $x(t) = 3 \sin 2.5t \qquad f = \frac{2.5}{2\pi}$

Sampling at 4 HZ is $> 2f$.

2. $x(t) = e^{-t}$ on $0 \le t \le 1$.

$$C_n = 1\int_0^1 (e^{-t}) e^{-jn(2\pi)t}\, dt$$

$$= \int_0^1 e^{-t(1+jn2\pi)}\, dt$$

$$= \left.\frac{e^{-t(1+jn2\pi)}}{-(1+jn2\pi)}\right|_0^1 =$$

$$\frac{-e^{-(1+jn2\pi)}+1}{1+jn2\pi}$$

$$x(t) = \sum_{n=-\infty}^{+\infty} \left(\frac{1-e^{-(1+jn2\pi)}}{1+jn2\pi}\right) e^{jn2\pi t}$$

3. i) $H(s) = \dfrac{s+3}{s^2+7s+1}$ $\qquad \dfrac{-7 \pm \sqrt{49-4}}{2} = -3.5 \pm \dfrac{\sqrt{45}}{2}$

$$h(t) = \mathcal{L}^{-1}\{H(s)\} = \frac{s+3}{(s+p_1)(s+p_2)}$$

where
$p_1 = +3.5 - \sqrt{45}/2$
$p_2 = +3.5 + \sqrt{45}/2$

$$= \frac{A}{s+p_1} + \frac{B}{s+p_2} \qquad A = \frac{-p_1+3}{p_2-p_1} = 0.5245$$

3 cont $\quad B = \dfrac{-p_2+3}{p_1-p_2} = 0.4255$

$$H(s) = \frac{0.5745}{s+p_1} + \frac{0.4255}{s+p_2}$$

$$h(t) = \left[0.5745\, e^{-p_2 t} + 0.4255\, e^{-p_2 t}\right] u(t)$$
$$= \left(0.5745\, e^{-6.8541t} + 0.4255\, e^{-.1459t}\right) u(t)$$

ii) We want

$$\mathcal{L}^{-1}\left[\frac{s+3}{s(s+p_1)(s+p_2)}\right] = \mathcal{L}^{-1}\left[\frac{A}{s} + \frac{B}{s+p_1} + \frac{C}{s+p_2}\right]$$

$$A = 3/p_1 p_2 = 3$$

$$B = \frac{3-p_1}{(-p_1)(p_2-p_1)} = -0.0838$$

$$C = \frac{3-p_2}{(-p_2)(p_1-p_2)} = -2.9162$$

$$\mathcal{L}^{-1}\left[\frac{3}{s} + \frac{-.0838}{s+p_1} + \frac{-2.9162}{s+p_2}\right]$$

$$= \left[3 - .0838 e^{-p_2 t} - 2.9162\, e^{-p_2 t}\right] u(t)$$

$$= \left(3 - .0838\, e^{-6.854t} - 2.9162\, e^{-.1459t}\right) u(t)$$

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4. i) $E[x] = \sum_{\text{all } x} x\, p(x) = \dfrac{(1+3+5+7+9+11)}{6}$

$= \underline{6}$

ii) $E[(x-\mu)^2] = \sum_{\text{all } x} (x-\mu)^2 p(x)$

$= \dfrac{(1-6)^2+(3-6)^2+(5-6)^2+(7-6)^2+(9-6)^2+(11-6)^2}{6}$

$= \dfrac{(-5)^2+(-3)^2+(-1)^2+(-1)^2+(3)^2+(5)^2}{6}$

$= \dfrac{70}{6} = 11.667$

$= E[x^2] - E[x]^2 = \dfrac{1}{6}\left[1+9+25+49+81+121\right] - 36$

$= 11.667$ ✓

5. Find α: $\frac{3}{2}\alpha \int_0^1 \int_0^1 (x-1)^2 y \, dy\, dx$

$= \frac{3}{2}\alpha \int_0^1 (x-1)^2 \left.\frac{y^2}{2}\right|_0^1 dx$

$= \frac{3}{4}\alpha \int_0^1 (x-1)^2 dx$

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$$= \frac{3}{4}\alpha \cdot \frac{(x-1)^3}{3}\Big|_0^1 = \frac{3}{4}\cdot\frac{\alpha}{3}\left[0+1\right]$$

$$= \frac{\alpha}{4} \quad \text{so} \quad \underline{\alpha = 4}$$

i) $$f_X(x) = \int_0^1 6(x-1)^2 y\, dy = 3(x-1)^2 y^2\Big|_0^1$$

$$= \underline{3(x-1)^2}$$

$$f_Y(y) = \int_0^1 6(x-1)^2 y\, dx = 2y\,(x-1)^3\Big|_0^1$$

$$= 2y(0+1) = 2y$$

$$f_X(x)\, f_Y(y) = 6y(x-1)^2 = f(x,y)$$

<u>yes</u>

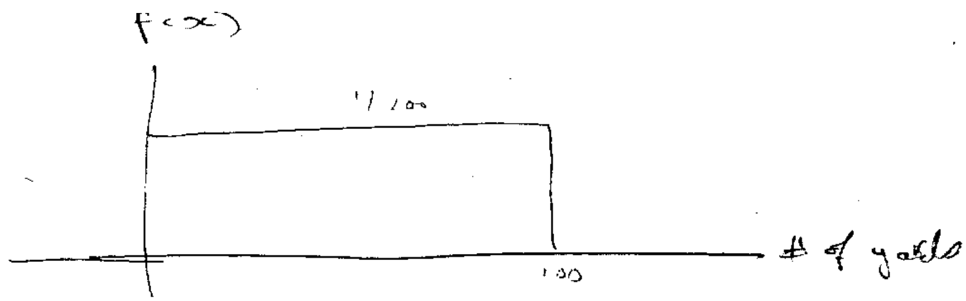
ii) $$P(X \geq .25) = \int_{.25}^{1} 3(x-1)^2\, dx = (x-1)^3\Big|_{.25}^{1}$$

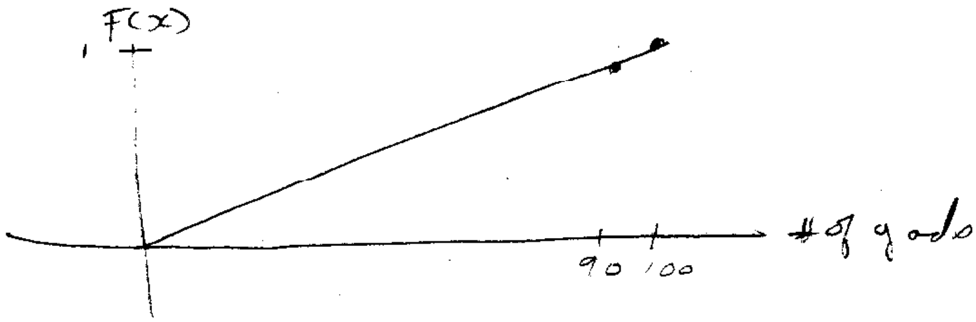
$$= .75^3 = \underline{0.421}$$

iii) $$F(x,y) = \int_0^x \int_0^y 6(x-1)^2 y\, dy\, dx = \int_0^x 3(x-1)^2 y^2\, dx$$

$$= (x-1)^3 y^2\Big|_0^x = \boxed{(x-1)^3 y^2 + y^2}$$

6.





7. $\begin{bmatrix}1\\1\\3\end{bmatrix} = \frac{1}{2}\begin{bmatrix}2\\2\\6\end{bmatrix}$ so only consider

$$\left\{ \begin{bmatrix}1\\1\\3\end{bmatrix}, \begin{bmatrix}1\\0\\0\end{bmatrix} \right\}$$

expressible vectors are of the form

$$\begin{bmatrix}\alpha+\beta\\ \alpha\\ 3\alpha\end{bmatrix}$$

$\begin{bmatrix}-7\\10\\30\end{bmatrix}$ is, $\begin{bmatrix}-7\\10\\29\end{bmatrix}$ and $\begin{bmatrix}-7\\10\\33\end{bmatrix}$ are not.

8.

a) $B\begin{bmatrix}1\\3\end{bmatrix} = \begin{bmatrix}8\\9\end{bmatrix}$ and $B\begin{bmatrix}0\\4\end{bmatrix} = \begin{bmatrix}4\\4\end{bmatrix}$

let $B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$

$$B\begin{bmatrix}1\\3\end{bmatrix} = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}\begin{bmatrix}1\\3\end{bmatrix} = \begin{bmatrix} b_{11}+3b_{12} \\ b_{21}+3b_{22} \end{bmatrix} = \begin{bmatrix}8\\9\end{bmatrix}$$

Similarly $\begin{bmatrix} 4b_{12} \\ 4b_{22} \end{bmatrix} = \begin{bmatrix}4\\4\end{bmatrix}$

$\underline{b_{12}=1}$, $\underline{b_{22}=1}$

$b_{11}+3b_{12}=8$ so $\underline{b_{11}=5}$

$b_{21}+3b_{22}=9$ $\underline{b_{21}=6}$

$$B = \begin{bmatrix} 5 & 1 \\ 6 & 1 \end{bmatrix}$$

b) using $\left\{\begin{bmatrix}10\\10\end{bmatrix}, \begin{bmatrix}0\\1\end{bmatrix}\right\}$

$$\begin{bmatrix} 10 & 0 \\ 10 & 1 \end{bmatrix}^{-1}\begin{bmatrix} 5 & 1 \\ 6 & 1 \end{bmatrix}\begin{bmatrix} 10 & 0 \\ 10 & 1 \end{bmatrix}$$

8 cont. $\frac{1}{10}\begin{bmatrix} 1 & 0 \\ -10 & 10 \end{bmatrix}\begin{bmatrix} 5 & 1 \\ 6 & 1 \end{bmatrix}\begin{bmatrix} 10 & 0 \\ 10 & 1 \end{bmatrix}$

$= \frac{1}{10}\begin{bmatrix} 5 & 1 \\ 10 & 0 \end{bmatrix}\begin{bmatrix} 10 & 0 \\ 10 & 1 \end{bmatrix} =$

$\frac{1}{10}\begin{bmatrix} 60 & 1 \\ 100 & 0 \end{bmatrix} = \begin{bmatrix} 6 & 0.1 \\ 10 & 0 \end{bmatrix}$

9. a. F — middle row always 0

b. F $\begin{bmatrix} 2 \\ 1 \\ 4 \end{bmatrix} = a\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} + b\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ $\left.\begin{matrix} a+b = 2 \\ b = 1 \\ a+b = 4 \end{matrix}\right\}$ inconsistent

c. T $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = a\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} + b\begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix} + c\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$

$\left.\begin{matrix} b+c=1 \\ a+2b=1 \\ a+2b+c=1 \end{matrix}\right\}$ $c=0$ $b=1$ $a=-1$

d. F middle row always 0

e. T. look at b_1, b_2, b_4:

$\left\{\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}\right\}$

$a\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} + b\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} + c\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$a+c=0$ $\underline{a=0.}$

$\underline{c=0}$

$a+b+c=0$ $\underline{b=0}$

only solution 0, 0, 0

9f. F $a\begin{bmatrix}1\\1\\1\end{bmatrix} + b\begin{bmatrix}0\\1\\1\end{bmatrix} + c\begin{bmatrix}1\\2\\2\end{bmatrix} = \begin{bmatrix}0\\0\\0\end{bmatrix}$

$$a + c = 0$$
$$\left.\begin{array}{l} a + b + 2c = 0 \\ a + b + 2c = 0 \end{array}\right\} \text{duplicate!}$$

$$a = -c$$
$$b = -a - 2c = a$$

any a, b, c where $b = a$
$c = -a$
work

g. F. Notice $b_7 = 2 * b_6$ so they are linearly dependent. Also $b_6 = b_4 + b_5$, so they are. There are only 2 independent vectors in the set.

h. T only need to try $\{b_4, b_5, b_1\}$

$a\begin{bmatrix}1\\1\\1\end{bmatrix} + b\begin{bmatrix}0\\1\\1\end{bmatrix} + c\begin{bmatrix}1\\0\\1\end{bmatrix} = \begin{bmatrix}0\\0\\0\end{bmatrix}$

$$a + c = 0 \qquad \underline{a = 0}$$
$$\left.\begin{array}{l} a + b = 0 \\ a + b + c = 0 \end{array}\right\} \underline{c = 0} \qquad \underline{b = 0}$$

only sol. $(0, 0, 0)$