Quiz #3 Solution -

Find the Step Response of

$$G(s) = \frac{3}{s^2 + 2s + 5}$$

Step 1 - PFE

We can immediately write

$$Output(s) = \frac{3}{s(s^2 + 2s + 5)}$$

and then recognize that

$$Output(s) = \frac{K1}{s} + \frac{K2s + K3}{s^2+2s+5}$$

We write it like this because the system poles are complex

Finding K1 is straightforward, K1= 3/5 = .6

Finding K2 & K3 requires some algebra, here is the common denominator method:

$$Output(s)= \frac{.6}{s} + \frac{K2s + K3}{s^2+2s+5} = \frac{.6(s^2+2s+5)}{s(s^2+2s+5)} + \frac{(K2s+K3)s}{s(s^2+2s+5)}$$

Or, more simply,

$$Output(s) = \frac{(.6+K2)s^2 + (1.2+K3)s + 3}{s(s^2+2s+5)}$$

But we know that this is just a simplification of the original Output(s) function, so

$$\frac{(.6+K2)s^2 + (1.2+K3)s + 3}{s(s^2+2s+5)} = \frac{3}{s(s^2+2s+5)}$$

Or,

.6+K2=0
1.2+K3=0
3=3

And therefore, K2=-.6
K3=-1.2

And we can say that

$$\text{Output}(s) = \frac{.6}{s} + \frac{-.6s - 1.2}{s^2+2s+5}$$

Step 2 - ILT

With a little bit of algebra, one can see that

$$\frac{.6}{s} + \frac{-.6s - 1.2}{s^2+2s+5} = \frac{.6}{s} + \frac{-.6*(s+1) - .6}{(s+1)^2 + 2^2}$$

$$= \frac{.6}{s} + (-.6)\frac{(s+1)}{(s+1)^2 + 2^2} + (-.3)\frac{2}{(s+1)^2 + 2^2}$$

We write it in this form to match enteries in our tables.

Finally, it is clear that

Output(t) = (.6 -.6 e^-t Cos[2t] -.3 e^-t Sin[2t]) u(t)