1. (a) Solve the following differential equation using Laplace Transform Methods. Assume all initial conditions are 0.

(b) Find the solution if the initial conditions are not all zero. Specifically, find x(t) when x(0)=0, x'(0)=.25.

$$\frac{d^2x(t)}{dt^2} + 4x(t) = t$$

Use 1) $\mathcal{L}\left[\frac{d^2x(t)}{dt^2}\right] = s^2 X(s) - sx(0) - x'(0)$

2) $\mathcal{L}[t] = 1/s^2$

$\therefore \quad \frac{dx^2(t)}{dt^2} + 4x(t) = t$

$\Rightarrow s^2X(s) - sx(0) - x'(0) + 4X(s) = \frac{1}{s^2}$

$X(s) = \frac{1}{s^2(s^2+4)} + \frac{sx(0) + x'(0)}{s^2+4}$

a) IC's 0 implies

$X(s) = \frac{1}{s^2(s^2+4)} = \frac{(1/4)}{s^2} + \frac{(-1/4)}{s^2+4}$

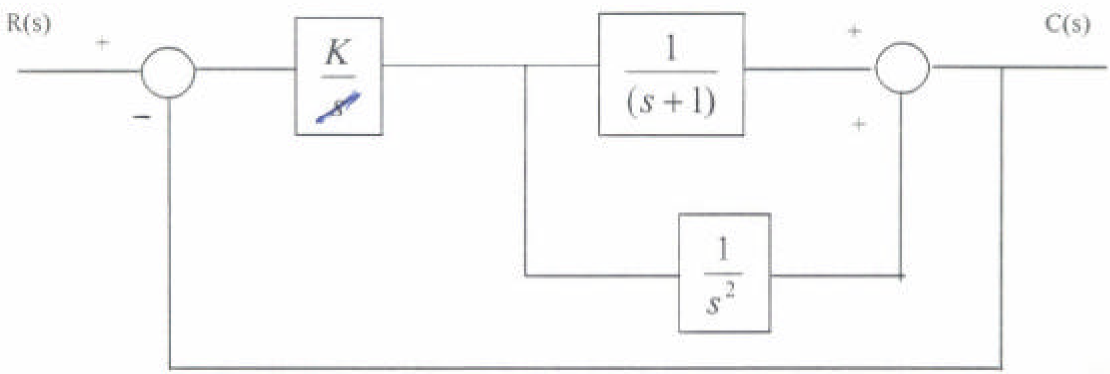
and $\boxed{x(t) = \left(\frac{1}{4}t - \frac{1}{8}\sin 2t\right)u(t)}$

b) $x(0) = 0$ and $x'(0) = .25$ implies

$X(s) = \frac{1}{s^2(s^2+4)} + \frac{.25}{s^2+4} = \underbrace{\frac{.25}{s^2} + \frac{-.25}{s^2+4}}_{\text{From above}} + \frac{.25}{s^2+4}$

$= \frac{.25}{s^2}$, and $\boxed{x(t) = .25\, t\, u(t)}$

2. Find the range of K for stability in the following system.



Parallel blocks: $\frac{1}{s+1} + \frac{1}{s^2} = \frac{s^2+s+1}{s^2(s+1)}$

Cascade blocks: $\frac{K(s^2+s+1)}{s^2(s+1)}$

Feedback form: $\frac{K(s^2+s+1)}{s^3+(K+1)s^2+Ks+K}$

Routh table:

s^3	1	K	
s^2	$K+1$	K	$\rightarrow K > -1$
s^1	$\frac{K^2}{K+1}$	0	$\rightarrow K > 0$
s^0	K		$\rightarrow K > 0$

$\boxed{\text{so } K > 0}$

3. (a) Find the single block (closed loop) transfer function for ***either*** problem (3-1) or (3-2).

(b) Draw the unit step response of your transfer function. Calculate the overshoot, settling time, and peak time.

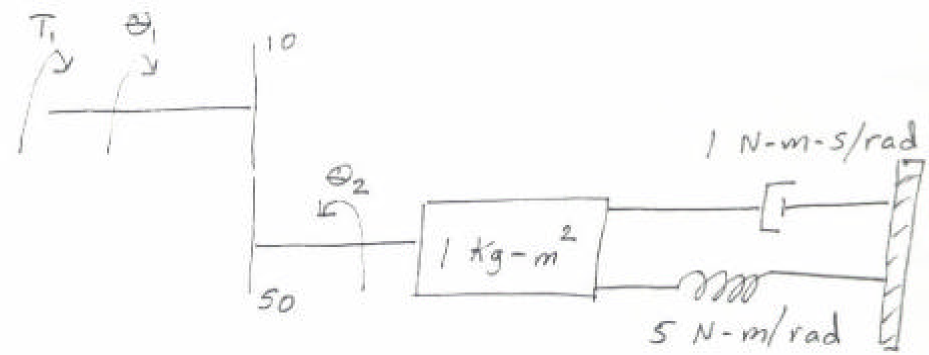


Figure 3-1. The transfer function of interest is θ2(s)/T(s)

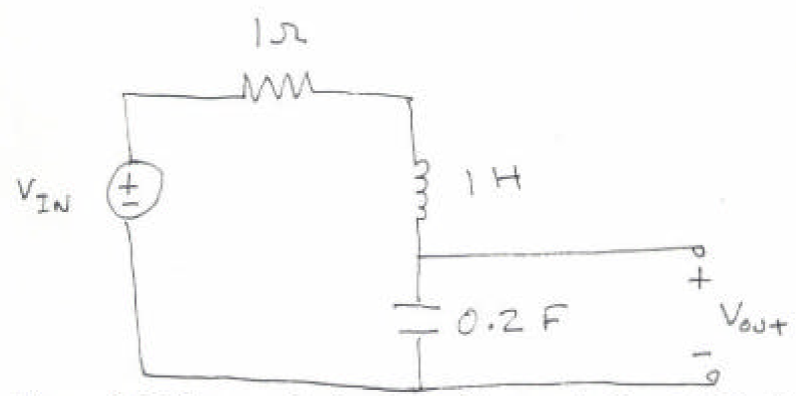
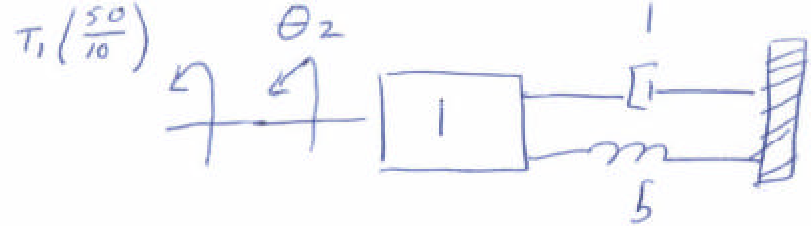
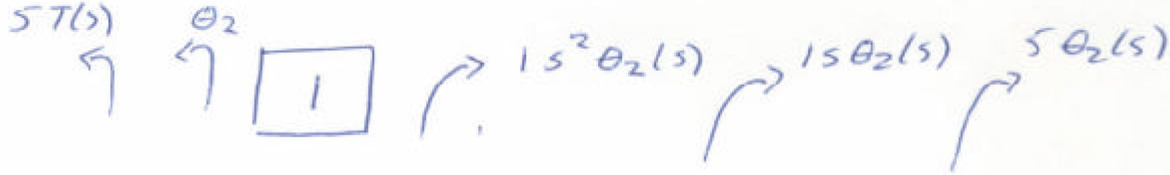


Figure 3-2. The transfer function of interest is Vout(s)/Vin(s)

a) 3-1

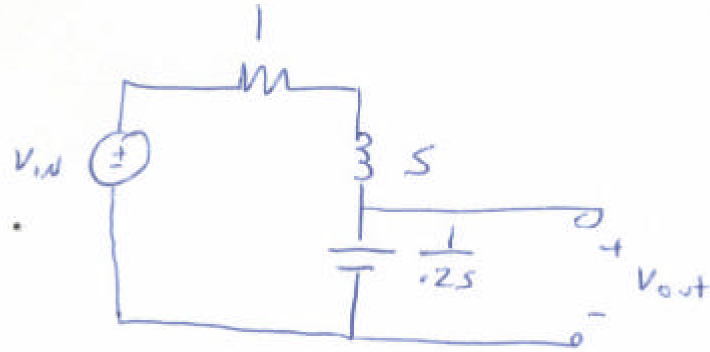




$$5T(s) = \theta_2(s)\left[s^2+s+5\right]$$

The Transfer Function is $\dfrac{5}{s^2+s+5}$

3-2



Voltage Divider

$$\frac{V_{out}(s)}{V_{IN}(s)} = \frac{\frac{1}{.2s}}{\frac{1}{.2s}+s+1}$$

$$= \frac{5}{s^2+s+5}$$

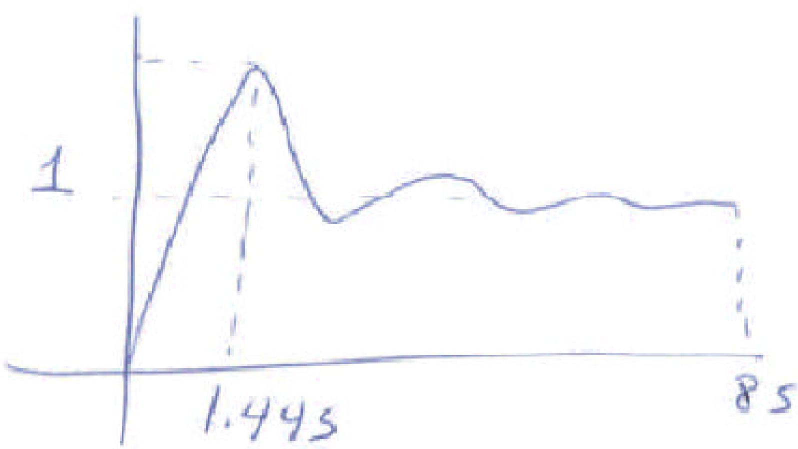
3 b) $\omega_n^2 = 5$ or $\omega_n = \sqrt{5}$

$2\zeta\omega_n = 1$ or $\zeta = .223$

$\Rightarrow OS = 48.74\%$

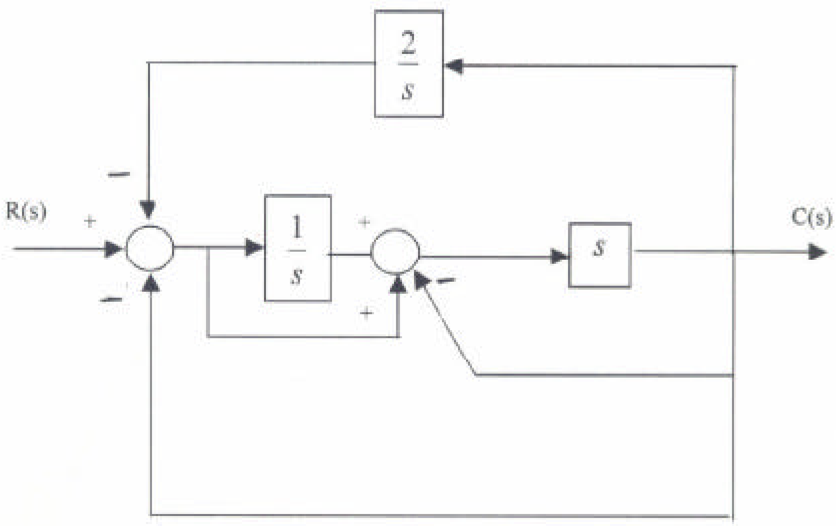
$T_p = 1.44s$

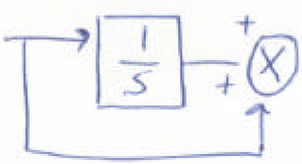
$T_s = 8s$



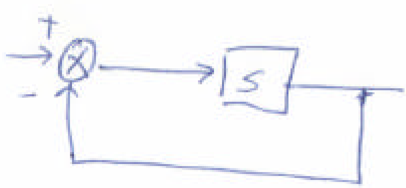
4. (a) Find the time domain output of the following system when excited by a step input. What is the steady state error?

(b) How do your answers for (a) change if the input is $e^{-3t}u(t)$?

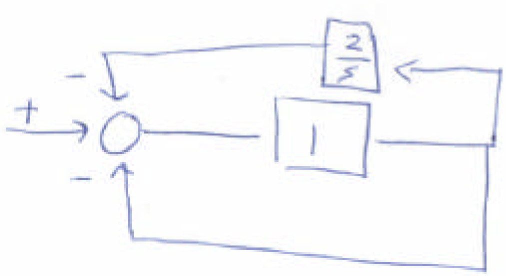


① 

Parallel form $\frac{1}{s}+1 = \frac{s+1}{s}$

② 

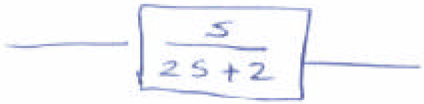
Feedback form $\frac{s}{s+1}$

③ 

$\frac{2}{s}$ and 1 in feedback

$$\frac{1}{1+\frac{2}{s}} = \frac{s}{s+2}$$

(4)



a) step Response

$$\mathcal{L}^{-1}\left[\frac{1}{s}\cdot\frac{s}{2s+2}\right] = \mathcal{L}^{-1}\left[\frac{\frac{1}{2}}{s+1}\right] = \frac{1}{2}e^{-t}u(t)$$

$$SSE = \lim_{s\to 0} \frac{1}{1+\frac{s}{2s+2}} = 1$$

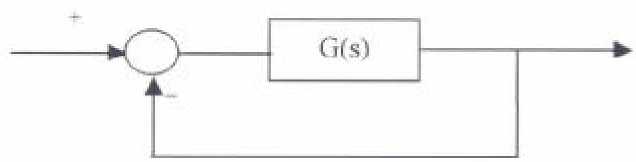
b) $e^{-3t}u(t)$ Response

$$\mathcal{L}^{-1}\left[\frac{1}{s+3}\cdot\frac{\frac{1}{2}s}{s+1}\right] = \mathcal{L}^{-1}\left[\frac{(3/4)}{s+3} + \frac{(-1/4)}{s+1}\right]$$

$$= \left(\frac{3}{4}e^{-3t} - \frac{1}{4}e^{-t}\right)u(t)$$

$$SSE = \lim_{s\to 0} \frac{s\,R(s)}{1+G(s)} = \lim_{s\to 0} \frac{\frac{s}{s+3}}{1+\frac{s}{2s+2}} = 0$$

5. In the following system, $G(s) = \dfrac{K_2}{s^2 + K_1 K_2 s + 1}$. Design (choose) values for K_1 and K_2 so that the steady state error due to a step input is less than .1 and the overshoot is less than 10%. Show that your choices for K_1 and K_2 are valid.



$$= \boxed{\frac{K_2}{s^2+K_1K_2s+(1+K_2)}}$$

SSE Criteria: $\lim_{s \to 0} \dfrac{1}{1+G(s)} = \dfrac{1}{1+K_2}$

$$\frac{1}{1+K_2} < 0.1 \Rightarrow K_2 > 9$$

OS Criteria: $OS < 10\% \Rightarrow \zeta \quad .5901$

Since $\omega_n^2 = K_2 + 1$

and $2\zeta\omega_n = K_1 K_2$

$$K_1 = \frac{2\zeta\omega_n}{K_2} = \frac{2\zeta\sqrt{K_2+1}}{K_2}$$

or $K_1 < \dfrac{1.18\sqrt{K_2+1}}{K_2}$

I choose $K_2 = 15$ and $K_1 = 0.3147$

(OVER)

Show Validity

Routh Table:

s^2	1	$1+K_2$
s^1	$K_1 K_2$	0
s^0	$1+K_2$	

so $K_1 K_2 > 0$

and $1+K_2 > 0$

both are true for

$K_1 = 0.3147$

$K_2 = 15.$