ECE 460 Final Exam Winter 1999

Name: KEY

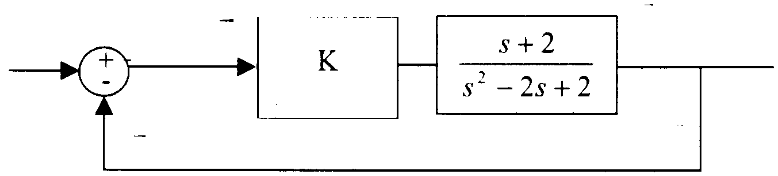
Honor Code:

Instructions:

- Complete the 5 problems in the allotted time.
- Use the space on the accompanying pages to work the problems. Do not use a bluebook. Attach additional worksheets if necessary.
- If you wish to have partial credit awarded for any of your incorrect answers **you must write clearly and legibly.** Explain your work in words, if necessary.
- Read the instructions provided with each problem. The problems all have multiple parts. The number of points allotted to each problem & each part is given.
- Don't Panic.

1. (25 points). There are parts a & b.

Consider the unity feedback system:



a) (17 points) **Draw the root locus**, using every sketching technique and refinement applicable to draw the locus as accurately as possible. Label all important quantities in your drawing. Use the following as a guideline:

i. (1 pt) Determine the open loop poles and zeros
ii. (1 pt) Find the portions of the real axis where the locus exists
iii. (3 pts) Find the jw-axis crossings (if any)
iv. (3 pts) Find the breakin/breakaway points (if any)
v. (3 pts) Find the asymptotes as K->infinity
vi. (3 pts) Find the angle of departure/arrival at all poles and zeros
vii. (3 pts) Sketch the locus.

i Poles $1 \pm j$ zeros -2

ii on axis $-\infty, -2$

$$\frac{K(s+2)}{s^2 + \cancel{2}s(K-2) + (2+2K)}$$

s^2	1	$2+2K$
s	$K-2$	0
s^0	$2+2K$	

$\boxed{K > 2}$

iii at $K=2$, $s = \pm\sqrt{6}\,j$

$$\frac{d}{ds}\left(\frac{s^2-2s+2}{s+2}\right)$$

$$= (s^2-2s+2)(-1)\frac{1}{(s+2)^2} + \frac{1}{s+2}(2s-2)$$

$$\frac{-s^2+2s-2+(2s^2+2s-4)}{(s+2)^2}$$

$$= \frac{s^2+4s-6}{(s+2)^2} \qquad \frac{-4\pm\sqrt{16-4(-6)}}{2}$$

$$= -2 \pm \frac{\sqrt{40}}{2}$$

$$= -2 \pm \sqrt{10}$$

iv $\underline{= -5.16 \text{ or } +1.16}$

V $\sigma = \frac{\sum p - \sum z}{n_p - n_z} = \frac{2+2}{1} = \underline{4}$

$\underline{\theta = 180^\circ}$

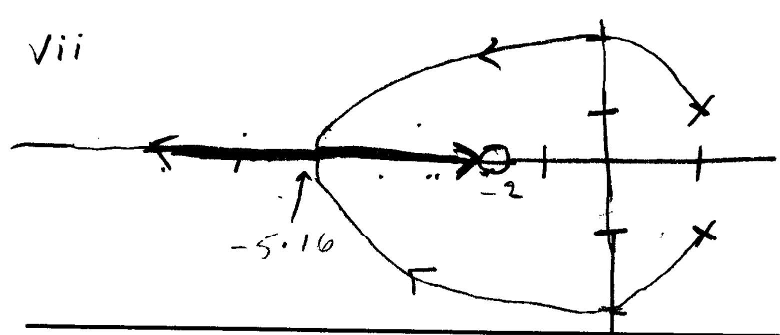
VI ouch!

angle at p_1

angle at p_2

angle at $z = 0^\circ$

vii



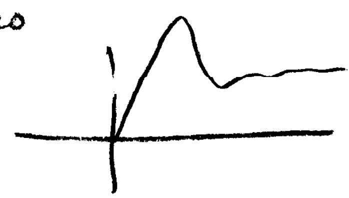
b. (8 points) Answer the following questions:

i) (1 point) What is the range of K for stability?
ii) (3 points) Does the locus pass through the point $-2+j*3$? What about $-2+j*3.1559$?
iii) (2 points) Sketch what the step response of the system look like when K=5.
iv) (2 points) What about when K=1?

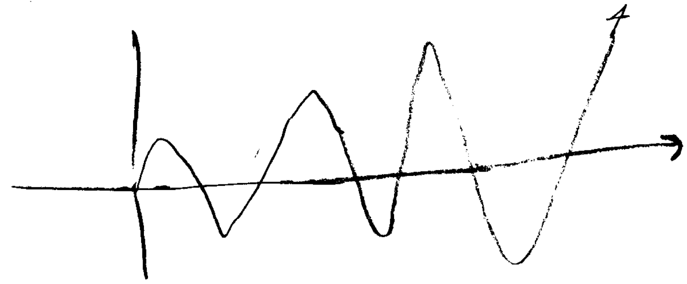
i $K > 2$

ii Via angle criterion, no ($\angle \neq \pm 180°$)
via angle criterion, yes ($\angle = 180°$)

iii system is underdamped w/ extra zero



iv system is unstable



2. (15 points total) There are parts a,b, c, and d.

a) (7 points) Sketch the asymptotic Bode magnitude plot for

$$G(s) = \frac{20(s+2)(s+20)}{s(s+10)(s+100)}$$

$\times \frac{2}{2} \times \frac{20}{20}$ $\frac{40}{1000} \times 20$

$\times \frac{10}{10} \times \frac{100}{100}$

Scale your graph appropriately. Label your axes.

Bpts 2, 10, 20, 100

Choose .1 → 1000

$$\frac{.8\left(\frac{s}{2}+1\right)\left(\frac{s}{20}+1\right)}{s\left(\frac{s}{10}+1\right)\left(\frac{s}{100}+1\right)}$$

$$20\log_{10}\left|\frac{.8}{.1}\right| = 20\log_{10}(8) \approx 18 \text{ dB}$$

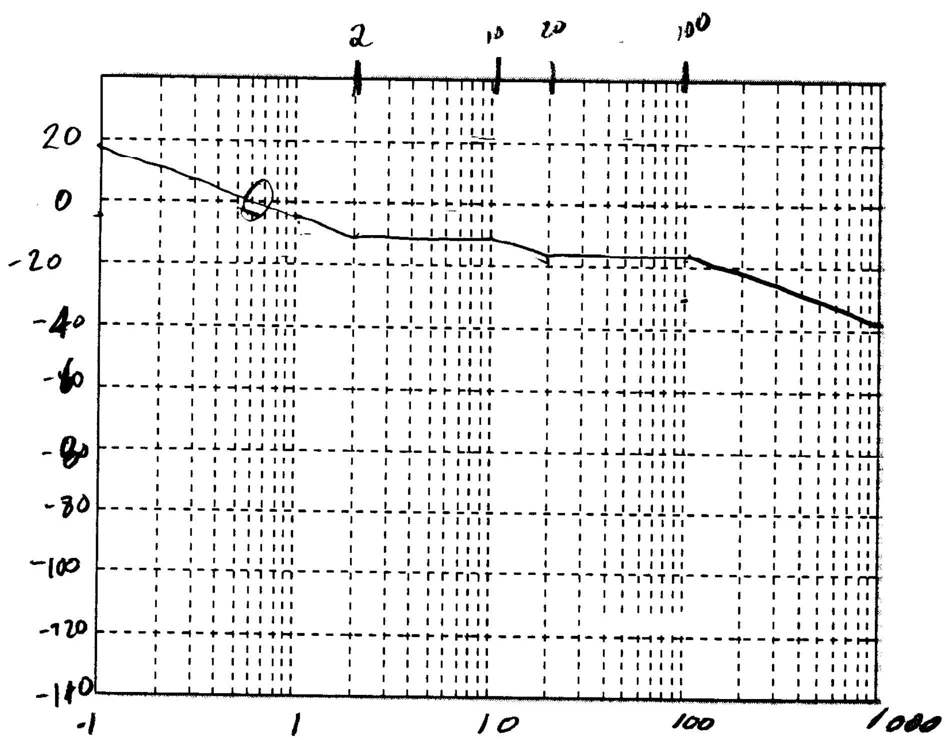
Starts −20 /dec

hits 2 → 0 dB/dec

hits 10 → −20 dB/dec

hits 20 → 0 dB/dec

hits 100 → −20 dB/dec

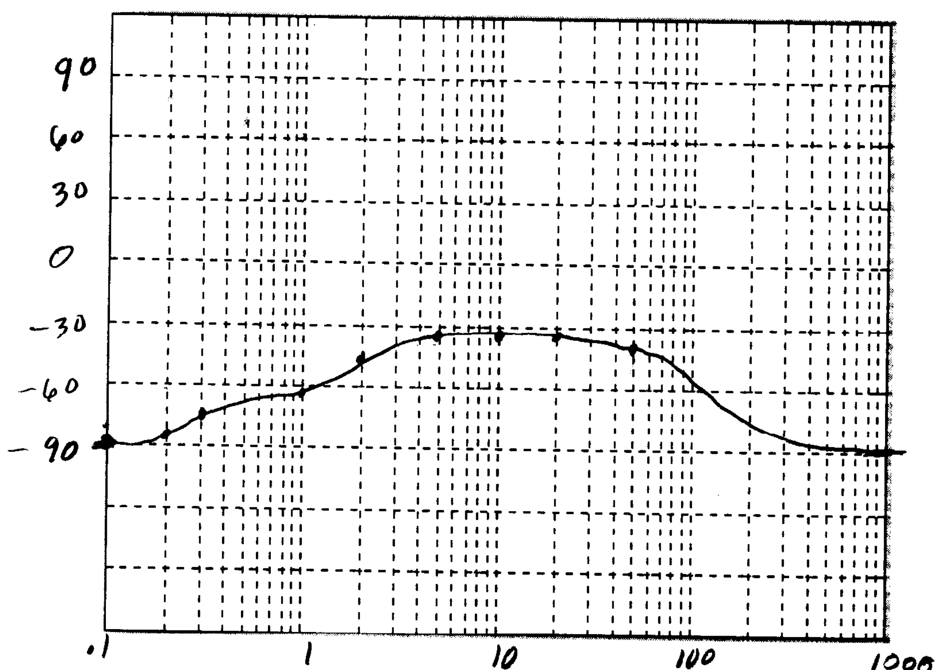


b) (4 points) Calculate the exact phase for w=0.1, 0.2, 0.5, 1, 2, 5, 10, 20, 50.

$\tan^{-1}(w/2) + \tan^{-1}(w/20) - 90 - \tan^{-1}(w/10) - \tan^{-1}(w/110)$

w	Phase
.1	-87.14
.2	-84.97
.5	-77.68
1	-66.85
2	-51.74
5	-37.19
10	-35.45
20	-35.45
50	-39.74

c) (3 points) Sketch the asymptotic Bode phase plot (use b. as a guide)



d) (1 point) Estimate the phase margin of your system.

102.32 from graph (110.52 act)

3. (10 points total) There are parts a and b.

a) (5 points) Write the following transfer function in state space form:

$$G(s) = \frac{1}{(s+3)(s+4)} = \frac{1}{s^2+7s+12} = \frac{C}{R}$$

$$\ddot{c} + 7\dot{c} + 12c = R$$

$$x = \begin{bmatrix} c \\ \dot{c} \end{bmatrix} \quad \dot{x} = \begin{bmatrix} \dot{c} \\ \ddot{c} \end{bmatrix} = \begin{bmatrix} x_2 \\ -12x_1 - 7x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ R \end{bmatrix}$$

$$\dot{X} = \underbrace{\begin{bmatrix} 0 & 1 \\ -12 & -7 \end{bmatrix}}_{A} x + \underbrace{\begin{bmatrix} 0 \\ 1 \end{bmatrix}}_{B} R$$

$$c = \underbrace{[1 \quad 0]}_{C^T} x \qquad D = 0$$

b) (5 points) Convert your answer to a) back to transfer function notation. Show all steps.

$$Y(s) = C(sI-A)^{-1}B + D$$

$$sI - A = \begin{bmatrix} s & -1 \\ 12 & s+7 \end{bmatrix}$$

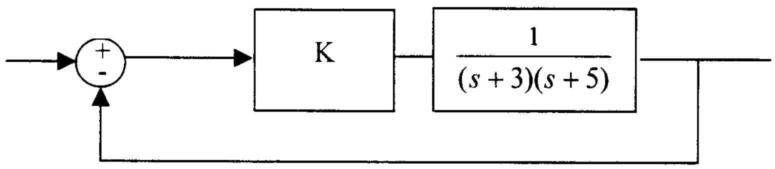
$$(sI-A)^{-1} = \frac{1}{s^2+7s+12}\begin{bmatrix} s+7 & 1 \\ -12 & s \end{bmatrix}$$

$$(sI-A)^{-1}B = \frac{1}{s^2+7s+12}\begin{bmatrix} 1 \\ s \end{bmatrix}$$

$$C(sI-A)^{-1}B = \underline{\frac{1}{s^2+7s+12}}$$

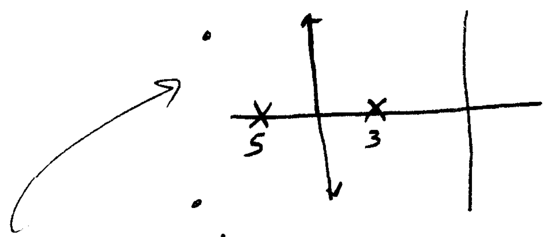
4. (25 points total) There are parts a,b,c,d,e, and f.

Consider the unity feedback system with



We wish to have the system operating with a settling time of 2/3 second and a percent overshoot of 1.5%.

a) (3 points) **Sketch** the root locus. Show that these criteria cannot be met by simply choosing K (i.e. show that the design point is not on the locus).



not on locus!

$T_s = 2/3$

$OS = .015$

$\zeta = .8007$

$\omega_n = 7.493$

desired pole

$-\zeta\omega_n + j\omega_n\sqrt{1-\zeta^2}$

$-6 \pm j\,4.4888$

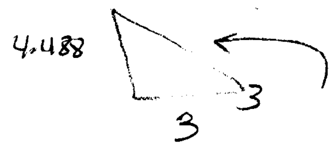
b) (3 points) Choose a compensator to meet the design criteria. Write a sentence defending your choice.

lead

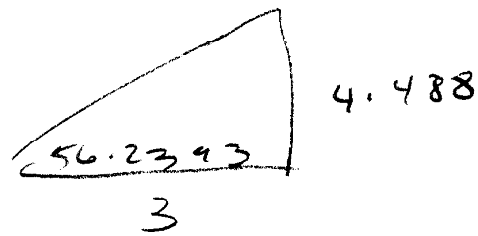
c) (10 points) Design your compensator. Specify the compensator's pole, zero and required gain.

$K \frac{s+5}{s+p}$

angle criterion



$\theta = 123.7602$



$\underline{p = 9}$

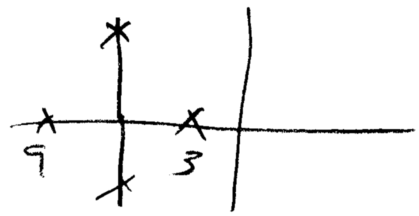
$s = -6 + j4.4888$

$K = |s+3||s+9| = 29.1493$

Controller $29.1493 \dfrac{s+5}{s+9}$

d) (3 points) Predict the performance of your system. If simulated, will it meet the criteria? Why or why not? Write a sentence – use a root locus sketch if necessary.

it'll be exact



e) (4 points) What is the steady state error due to a step input of your system? Design a compensator to reduce that error by half.

.4809

how 'bout

$$\frac{s+.01}{s+.005}$$

f) (2 points) Write a sentence on how your compensator in e) will effect the transient response.

almost cancel - ~~[illegible]~~ negligible

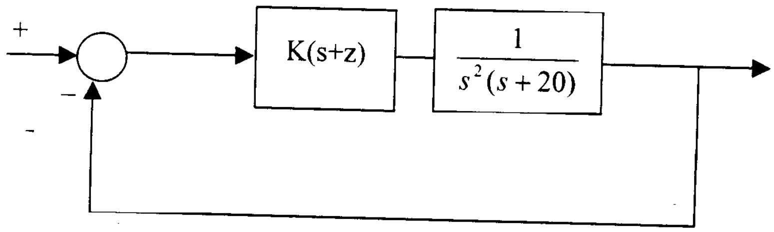
~~if anything~~ OS ↑

~~T4~~

Ts lengthed due to gradual rise!

5. (25 points) There are parts a,b,c, and d.

(10 points) a) Choose the values of K and z to meet the transient specifications of 16.3% OS and 8s settling time.
(5 points) b) What is the steady state error due to a unit step, a unit ramp, and a unit parabola?
(5 points) c) Discuss the validity of your second order approximation.
(5 points) d) Give the exact overshoot and settling time if z=0 and K=200.



denominator $\frac{K(s+z)}{s^3+20s^2+Ks+Kz}$

$$s^3+20s^2+Ks+Kz = (s+p)(s^2+s+1)$$

$$s^3+20s^2+Ks+Kz = s^3+s^2(p+1)+s(p+1)+p$$

$P = 19$

a) $K = 20$, $z = \frac{19}{20}$

b) $SSE_{step} = \lim_{s\to 0} \frac{1}{1+G(s)} = 0$

$SSE_{ramp} = 0$

$SSE_{parabola} = \frac{1}{s^2G(s)} = \frac{1}{\frac{20\cdot 19/20}{20}} = 20/19$

c) OS probably too high due to zero at .95
third pole negligible.

d) $\frac{200}{s(s+20)} \rightarrow \frac{200}{s^2+20s+200}$

$\omega_n = \sqrt{200}$
$2\zeta\omega_n = 20$, $\zeta = .7071$
$T_s = 0.4$, $OS = 4.32\%$