ECE 460 Midterm Exam Winter 2001

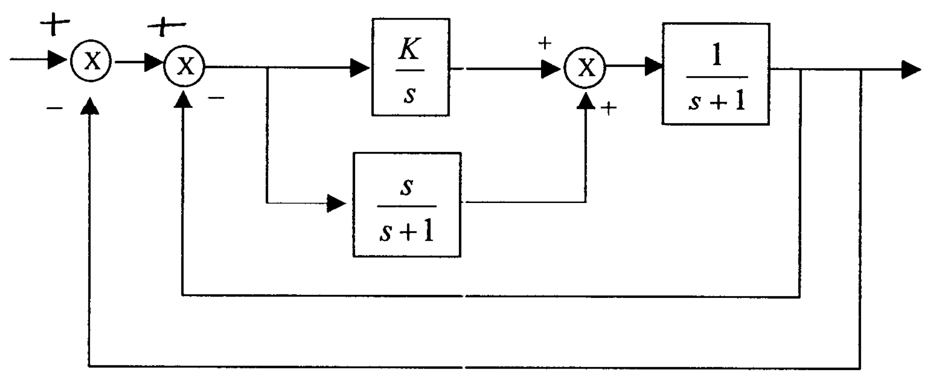
Name:

Honor Code: KEY

Instructions:

- Use the space on the accompanying pages to work the problems. Do not use a bluebook. Attach additional worksheets if necessary.
- If you wish to have partial credit awarded for any of your incorrect answers **you must write clearly and legibly.** Explain your work in words, if necessary.
- Read the instructions provided with each problem.
- Don't Panic.

1. [20 points] Find the range of K for stability in the following system



Parallel: $\frac{K}{s} + \frac{s}{s+1} = \frac{K(s+1) + s^2}{s+1}$

Cascade: $\frac{K(s+1)+s^2}{(s+1)^2}$ Feedback: $\frac{K(s+1)+s^2}{(s+1)^2+K(s+1)+s^2}$

Feedback(2): $\frac{K(s+1)+s^2}{(s+1)^2+K(s+1)+s^2+K(s+1)+s^2} = \frac{s^2+Ks+K}{s^3+4s^2+s(1+2K)+2K}$

(OVER)

Stability:

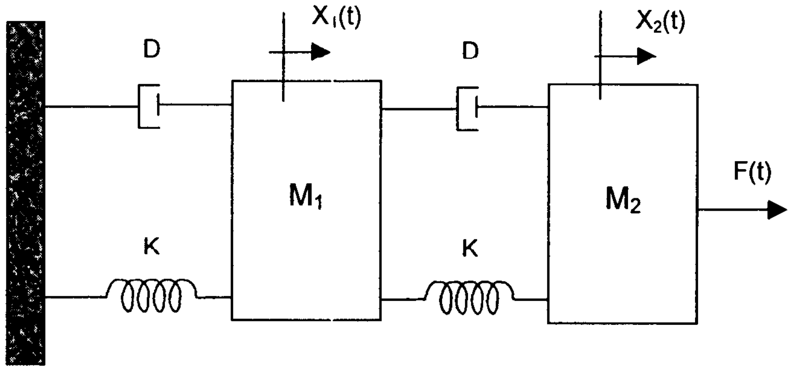
$$
\begin{array}{c|ccc}
s^3 & 1 & 1+2K & \\
s^2 & 4 & 2K & \\
s^1 & \dfrac{\begin{vmatrix} 1 & 1+2K \\ 4 & 2K \end{vmatrix}}{-4} = \dfrac{3K+2}{2} & 0 & \Rightarrow \quad K > -\dfrac{2}{3} \\
s^0 & 2K & 0 & \Rightarrow \quad K > 0
\end{array}
$$

Stability for $K > 0$

2.

(a) [20 points] Find the Transfer Function $X_1(s)/F(s)$

(b) [10 points] What is the order of this system? Discuss the stability when $D=1$, $K=1$ and $M_1 = M_2 = 1$.



M1

$$X_1(s)\left[M_1 s^2 + 2Ds + 2K\right] + X_2(s)\left(-Ds - K\right) = 0$$

M2

$$X_1(s)\left[-Ds - K\right] + X_2(s)\left[M_2 s^2 + 2Ds + 2K\right] = F(s)$$

or $$X_2(s) = X_1(s)\left[\frac{M_1 s^2 + 2Ds + 2K}{Ds + K}\right]$$

Then

$$F(s) = X_1(s)\left[-Ds - K + \frac{(M_2 s^2 + 2Ds + 2K)(M_1 s^2 + 2Ds + 2K)}{Ds + K}\right]$$

and $$\boxed{\frac{X_1(s)}{F(s)} = \frac{Ds + K}{(M_1 s^2 + 2Ds + 2K)(M_2 s^2 + 2Ds + 2K) - (Ds + K)^2}}$$

For The given values,

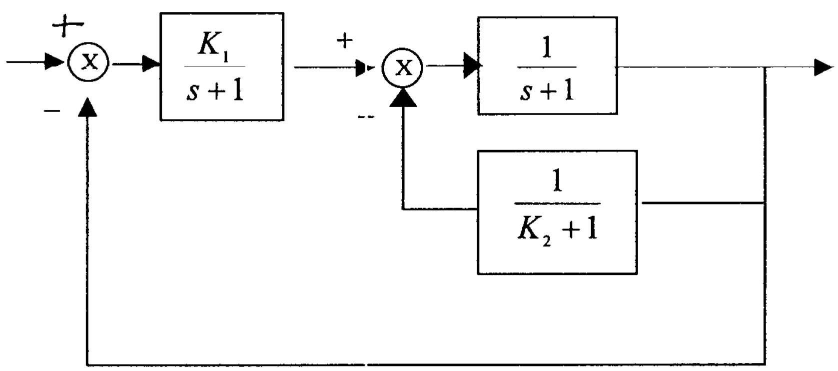
$$\frac{X_1(s)}{F(s)} = \frac{s+1}{s^4 + 3s^3 + 4s^2 + 2s + 1}$$

s^4	1	4	1
s^3	3	2	
s^2	$\dfrac{\begin{vmatrix} 1 & 4 \\ 3 & 2 \end{vmatrix}}{-3} = \dfrac{10}{3}$	1	
s^1	$\dfrac{\begin{vmatrix} 3 & 2 \\ 10/3 & 1 \end{vmatrix}}{-10/3} = \dfrac{11}{10}$		
s^0	1		

No sign changes $\Rightarrow$ stable

3.

(a) [10 points] Simplify the following system to a single block.
(b) [10 points] Find all second order parameters and sketch the step response when K1 = 1 and K2 = 2. Show that the system is stable.
(c) [10 points] Given that K1 = 1, find the range of K2 for stability.



(a) Feedback:

$$\frac{\frac{1}{s+1}}{1+\left(\frac{1}{s+1}\right)\left(\frac{1}{K_2+1}\right)} = \frac{K_2+1}{s(K_2+1)+(2+K_2)}$$

Cascade

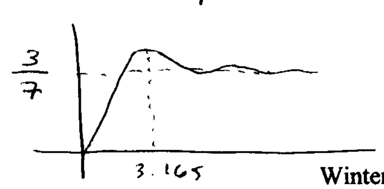
$$\frac{K_1(K_2+1)}{s^2(K_2+1)+s(2K_2+3)+(2+K_1+K_2+K_1K_2)}$$

(b) $K_1 = 1$, $K_2 = 2$

$$\frac{3}{3s^2+7s+7} = \frac{1}{s^2+\frac{7}{3}s+\frac{7}{3}}$$

$\omega_n = \sqrt{7/3}$ $\quad 2\zeta\omega_n = 7/3$, $\zeta = 0.76$

$OS = 2.54\%$ $\quad T_s = 3.44s$ $\quad T_p = 3.165s$



$s^2 + \frac{7}{3}s + \frac{7}{3}$ is stable:

s^2	1	$7/3$
s^1	$7/3$	
s^0	$7/3$	

(c) given $K_1 = 1$ $\qquad G(s) = \dfrac{K_2 + 1}{s^2(K_2+1) + s(2K_2+3) + (3+2K_2)}$

s^2	K_2+1	$3+2K_2$
s^1	$2K_2+3$	
s^0	$3+K_2$	

Need 0 sign changes

all pos

$K_2 > -1$

$2K_2 + 3 > 0 \quad K_2 > -\frac{3}{2}$

$3 + K_2 > 0 \quad K_2 > -3$

or

all neg

$K_2 < -1$

$K_2 < -3/2$

$K_2 < -3$

$\underline{K_2 > -1}$ or $\underline{K_2 < -3}$

4. [20 points] Solve the following ODE for $x(t)$. Assume all IC's are 0.

$$\frac{d^2x(t)}{dt^2}+2\frac{dx(t)}{dt}+x(t)=\cos(1t)$$

$$X(s)\left[s^2+2s+1\right] = \frac{s}{s^2+1}$$

$$X(s) = \frac{s}{(s^2+1)(s+1)^2}$$

$$= \frac{As+B}{s^2+1} + \frac{C}{(s+1)^2} + \frac{D}{s+1}$$

$$\underline{C = -1/2}$$

$$s = (As+B)(s+1)^2 + C(s^2+1) + D(s+1)(s^2+1)$$

$$= s^3(A+D) + s^2(2A+B+C+D) + s(A+2B+D) + (B+C+D)$$

$$\Rightarrow \quad A=0 \quad D=0 \quad B=1/2$$

$$X(s) = \frac{1/2}{s^2+1} + \frac{-1/2}{(s+1)^2}$$

$$x(t) = \left(\frac{1}{2}\sin(t) - \frac{1}{2}te^{-t}\right)u(t)$$