8-1

a) Cannot be a locus. It violates Rule 2 (symmetry), Rule 3 (real-axis existence)
b) Can be a locus.
c) Can be a locus.
d) Can be a locus.
e) Cannot be a locus. It violates Rule 2 (symmetry), Rule 3 (real-axis existence)
f) Can be a locus.
g) Cannot be a locus. It violates Rule 2 (symmetry), Rule 3 (real-axis existence)
h) Can be a locus.

8-2

For this problem (and for many of the problems in the rest of this course), your answers can be **verified** using matlab:

```
rlocus(num,den)
```

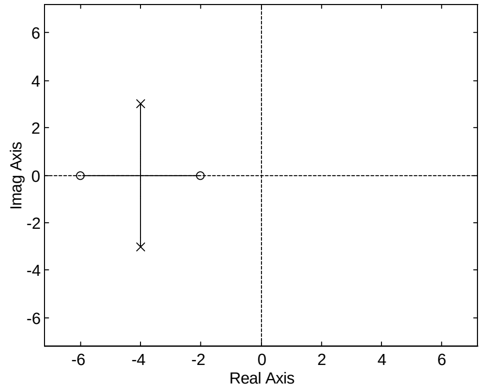
Of course, you should be able to sketch these loci by hand using the rules only.

Note also, we are sketching – which means we investigate rules 1-5 only. The other rules are applied only when a refinement is specifically asked for by the problem.

a)

OL zeros: -2,-6
OL poles: -3-4j, -3+4j

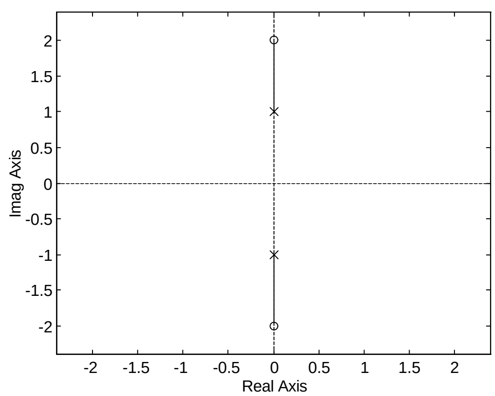
Rule 1 – There are two branches
Rule 2 – The locus is symmetric about the real axis
Rule 3 – The locus is on the axis on [-2…-4] and [-4…-6]. In other words, it is on the axis on [-2…-6]
Rule 4 – The locus starts at the OL poles (-3-4j and –3+4j) and ends at the zeros (-2, -6)
Rule 5 – Doesn’t apply – Number of finite OL poles = Number of finite OL zeros (2=2)



b)

OL zeros: -2j, +2j
OL poles: -j, +j

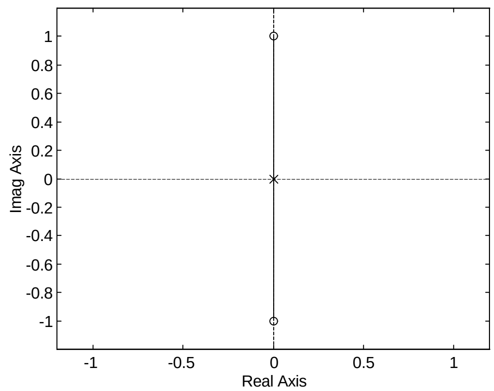
Rule 1 – There are 2 branches
Rule 2 – The locus is symmetric about the real axis
Rule 3 – The locus never on the real axis
Rule 4 – The locus starts at the OL poles (-j,+j) and ends at the OL zeros (-2j, +2j)
Rule 5 – Doesn't apply



c)

OL zeros: -j,+j
OL poles: 0,0

Rule 1 – There are two branches
Rule 2 – The locus is symmetric about the real axis
Rule 3 – The locus on the real axis at s=0 only
Rule 4 – The locus starts at the OL poles (0,0) and ends at the OL zeros (-j,+j)
Rule 5 – Doesn't apply (number of finite OL poles = number of finite OL zeros)



8-3

Clearly, the open loop poles are at –1 and 0
Also, the open loop zeros are at 1+j and 1-j

Therefore, the transfer function is

$$\frac{(s-1-j)(s-1+j)}{(s+1)(s)} = \frac{s^2-2s+2}{s(s+1)}$$

Rule 1 – There are two branches
Rule 2 – The locus is symmetric about the real axis
Rule 3 – The locus is on the axis between –1 and 0
Rule 4 – The locus starts at the OL poles (-1 and 0) and ends at the OL zeros (1+j & 1-j)
Rule 5 – Doesn't apply

The breakaway point(s) is found via rule 6 or 6'
Rule 6 –

$$\frac{d}{ds}\left[\frac{-1}{G(s)H(s)}\right] = \frac{-d}{ds}\left[\frac{s^2+s}{s^2-2s+2}\right] = \frac{-(-3s^2+4s+2)}{(s^2-2s+2)^2}$$

This function has zeros when $-3s^2+4s+2 = 0$, or at s=-0.3874 or 1.7208

The breakaway point that is valid here is -0.3874. Notice that 1.7208 is a solution to the quadratic, but since the locus is not on the axis at 1.7208 it is discarded.

Rule 6' – alternate formulation of the same equation

$$\frac{1}{s+0}+\frac{1}{s+1}=\frac{1}{s+(-1-j)}+\frac{1}{s+(-1+j)}$$

Simplifying,

$$\frac{2s+1}{s^2+s}=\frac{2s-2}{s^2-2s+2}$$

Cross Multiplying,

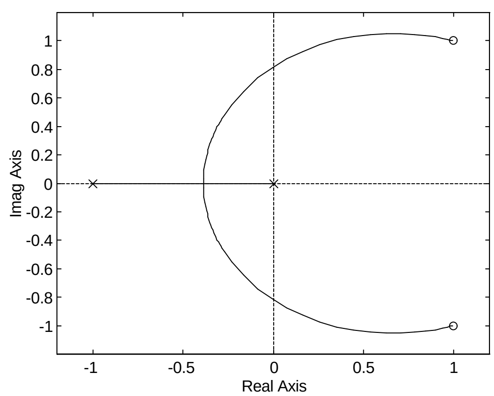
$$2s^3-3s^2+2s+2=2s^3-2s$$

Or,

$$-3s^2+4s+2=0$$

and $s=-0.3874$, as before

Finally, the locus looks like:



8-13

OL poles: 0,-4,-8
OL zeros: No finite zeros, therefore there are 3 zeros at infinity

Rule 1 – The locus has 3 branches
Rule 2 – The locus is symmetric about the real axis
Rule 3 – The locus is on the axis between [0…-4] and [-8…-infinity]
Rule 4 – The locus starts at the open loop poles (0,-4,-8) and ends at the zeros at inifinity. How?

Rule 5 – The locus approaches infinite zeros with the following asymptotes:

Theta = (2k+1)*pi/(# of finite poles - # of finite zeros)
=(2k+1)*pi/(3) = 60°, 180°, 300°

sigma = (sum(poles)-sum(zeros))/(# of finite poles - # of finite zeros)
= (0+-4+-8)/(3) = -4

Rule 6 – Breakaway points are found as

$$\frac{d}{ds}\left[\frac{-1}{G(s)H(s)}\right] = \frac{-d}{ds}\left[\frac{s(s+4)(s+8)}{1}\right] = 3s^2 + 24s + 32$$

$$\text{And therefore we find s} = -1.69 \text{ and - } 6.3094$$

Rule 6' – The alternate method for breakaway points says

$$\frac{1}{s+0} + \frac{1}{s+4} + \frac{1}{s+8} = 0$$

Simplifying,

$$\frac{3s^2 + 24s + 32}{s^3 + 12s^2 + 32s} = 0$$

And we have the same answer as before

Notice that both of these produce two roots. It turns out that only s=-1.69 is valid.

Rule 7 – jw axis crossings may be determined by solving the characteristic equation with s=jw

$$1 + KGH = 0$$

$$1 + K\frac{1}{s(s+4)(s+8)}$$

evaluated when $s = jw$,

$$1 + K\frac{1}{j(32w - w^3) - 12w^2} = 0$$

Simplifying,

$$j(32w - w^3) - 12w^2 = -K$$

Equating real & imaginary parts,

$$32w - w^3 = 0, \text{ or } w = sqrt(32) = 5.65$$

$$and\ \mathrm{K} = 12w^2 = 384$$

Rule 7' – Alternate method, using a Routh table

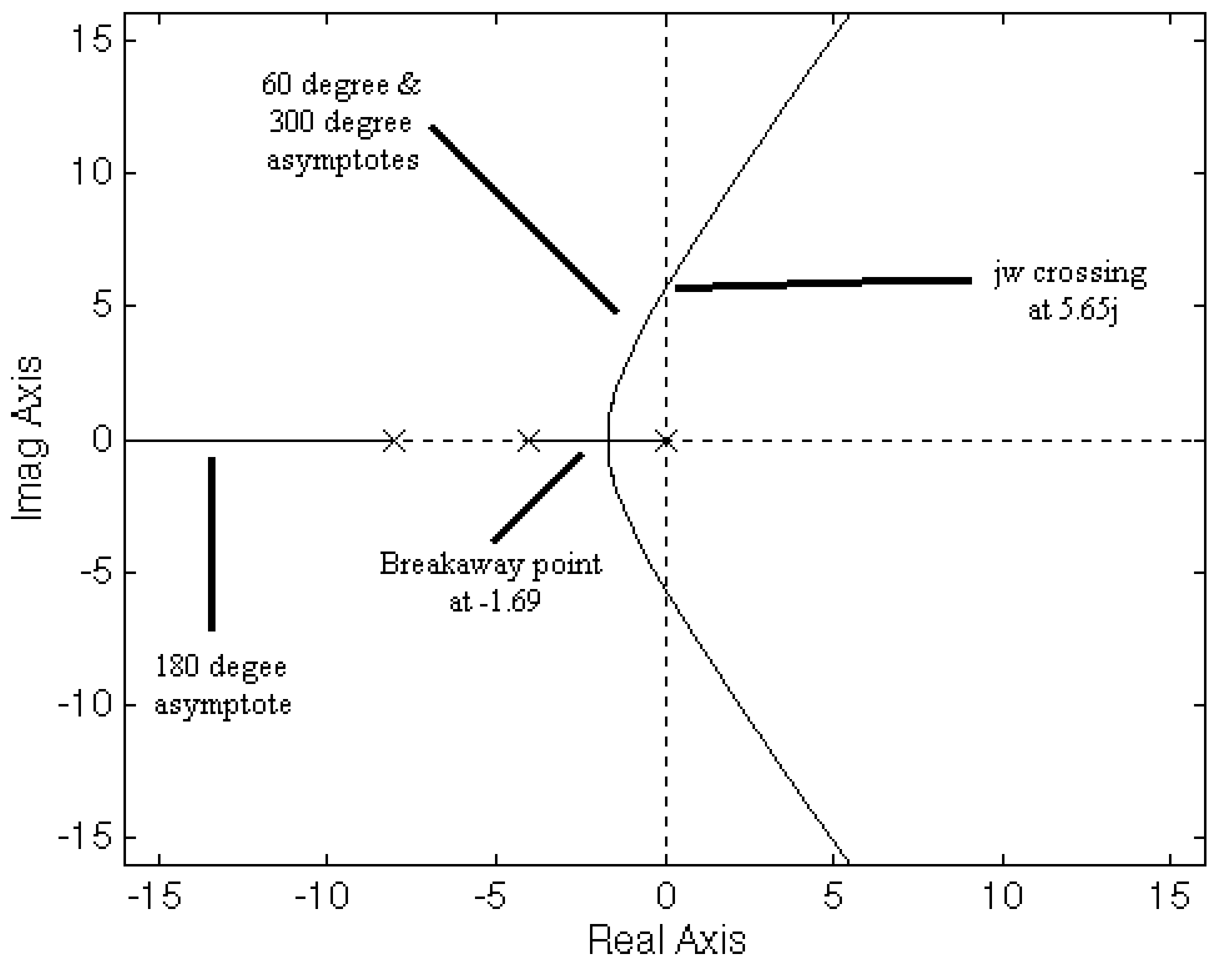
The closed loop transfer function is

$$\frac{K}{s(s+4)(s+8)+K} = \frac{K}{s^3 + 12s^2 + 32s + K}$$

and the Routh table is

s3	1	32	
s2	12	K	
s1	(K-384)/12		0
s	K		

a row of all zeros occurs when K=0 (as expected – there is an open loop pole on the jw axis) and when K=384. At this point $s^3+12s^2+32s+K = s^3+12s^2+32s+384$ and the roots are at s=5.65j and –5.65j



60 degree &
300 degree
asymptotes
jw crossing
at 5.65j
Breakaway point
at -1.69
180 degee
asymptote
Imag Axis
Real Axis