

## Chapter 4, Problem 23

The poles of the system are located at  $-5, -.6 \pm j 1.57$

Clearly, the added pole is more than 5 times farther from the Imaginary axis than the dominant second order poles.

We can approximate this as purely second order.

The dominant poles are at  $-.6 \pm j 1.57$ , which makes

Therefore,

$\omega_n=1.6820$  and  $\zeta=.3567$

The relevant quantities are then easily found:

OS=30%

$T_s=6.64$  s

$T_p=2$  s

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Chapter 4, Problem 24

a) From the graph, it is clear that the response reaches 63% of it's peak value at  $\tau=.025$  seconds (approx), which makes  $a=1/\tau = 40$ .

The graph settles at 2 instead of 1, so the transfer function is  $2 * a/(s+a)$ , or

$$2 * 40 / (s+40)$$

You can check this is correct by using the Laplace transform properties:

$$\lim_{s \rightarrow 0} s \text{ Output}(s) = \lim_{t \rightarrow \infty} \text{Output}(t)$$

or

$$\lim_{s \rightarrow 0} \frac{s * 2 * 40}{s(s+40)} = 2, \text{ which is where Output}(t) \text{ settles}$$

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b) From the graph, it is easy to measure that the maximum value is  $\sim 13.75$ . The final value is  $\sim 11$ . Therefore, the

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overshoot is  $(13.75-11)/11 = 25\%$ .

Also, the peak time is at  $\sim 1$  second.

Using these values, we can calculate  $\zeta=0.4$  by eq (4.39) and  $\omega_n=3.43$ .

Therefore the denominator of the transfer function is  $(s^2 + 2*\zeta*\omega_n s + \omega_n^2) = (s^2 + 2.744 s + 11.76)$ .

Since the curve settles at 11, we can write the transfer function as

$$G(s) = \frac{11 * (11.76)}{s^2 + 2.744 s + 11.76} = \frac{129.36}{s^2 + 2.744 s + 11.76}$$