$$\frac{d^2x}{dt^2} + 10\frac{dx}{dt} + 21x = 8u(t)$$

Step 1 Laplace Transform [assume $x(0) = \dot{x}(0) = 0$]

$$\mathcal{L}\left[\frac{d^2x}{dt^2}\right] = s^2X(s) - s\,x(0) - \dot{x}(0) = s^2X(s)$$

$$\mathcal{L}\left[\frac{dx}{dt}\right] = sX(s) - x(0) = sX(s)$$

$$\mathcal{L}[x] = X(s)$$

$$\mathcal{L}[u(t)] = 1/s$$

From Your Tables

so $$\frac{dx^2}{dt^2} + 10\frac{dx}{dt} + 21x = 8u(t)$$

transforms to $s^2X(s) + 10sX(s) + 21X(s) = 8/s$

or $$X(s) = \frac{8}{s(s^2+10s+21)}$$

Step 2 PFE

$$\frac{8}{s(s^2+10s+21)} = \frac{8}{s(s+7)(s+3)} = \frac{K_1}{s} + \frac{K_2}{s+7} + \frac{K_3}{s+3}$$

$$K_1 = \lim_{s\to 0} \frac{8}{(s+7)(s+3)} = 8/21$$

$$K_2 = \lim_{s\to -7} \frac{8}{s(s+3)} = 8/28$$

$$K_3 = \lim_{s\to -3} \frac{8}{s(s+7)} = -8/12$$

a) $$\frac{2s}{(s+3)(s+7)} = \frac{K_1}{s+3} + \frac{K_2}{s+7}$$

$$K_1 = \lim_{s \to -3} \frac{2s}{(s+7)} = \frac{-6}{4} = -1.5$$

$$K_2 = \lim_{s \to -7} \frac{2s}{(s+3)} = \frac{-14}{-4} = +3.5$$

so $$\frac{2s}{(s+3)(s+7)} = \frac{-1.5}{s+3} + \frac{3.5}{s+7}$$

and $$f(t) = -1.5e^{-3t}u(t) + 3.5e^{-7t}u(t)$$

b) $$\frac{2s}{(s+3)(s+7)(s+10)} = \frac{K_1}{s+3} + \frac{K_2}{s+7} + \frac{K_3}{s+10}$$

$$K_1 = \lim_{s \to -3} \frac{2s}{(s+7)(s+10)} = \frac{-6}{(4)(7)} = \frac{-3}{14}$$

$$K_2 = \lim_{s \to -7} \frac{2s}{(s+3)(s+10)} = \frac{-14}{(-4)(3)} = \frac{7}{6}$$

$$K_3 = \lim_{s \to -10} \frac{2s}{(s+3)(s+7)} = \frac{-20}{(-7)(-3)} = \frac{-20}{21}$$

so $$\frac{2s}{(s+3)(s+7)(s+10)} = \frac{-3/14}{s+3} + \frac{7/6}{s+7} + \frac{-20/21}{s+10}$$

and $$f(t) = -3/14\, e^{-3t}u(t) + 7/6\, e^{-7t}u(t) + -20/21\, e^{-10t}u(t)$$

c) $$\frac{2s}{s^2+10s+50} = \frac{2s}{(s+3)^2+5^2}$$

$$= (2)\frac{(s+5)}{(s+5)^2+5^2} + (-2)\frac{5}{(s+5)^2+5^2}$$

$$f(t) = 2e^{-5t}\sin 5t\ u(t) - 2e^{-5t}\cos 5t\ u(t)$$

d) 13 a $$\frac{dx}{dt} + 7x = 5\cos 2t$$

$$\Rightarrow \quad s\,X(s) + 7X(s) = 5\left(\frac{s}{s^2+4}\right)$$

$$X(s) = \frac{5s}{(s^2+4)(s+7)}$$

$$= \frac{K_1 s + K_2}{s^2+4} + \frac{K_3}{s+7}$$

$$K_3 = \lim_{s \to -7} \frac{5s}{s^2+4} = \frac{-35}{53}$$

Common denominator:

$$\frac{K_1 s + K_2}{s^2+4} + \frac{(-35/53)}{s+7} =$$

13a cont

$$= \frac{(K_1 s + K_2)(s+7) + (-35/53)(s^2+4)}{(s+7)(s^2+4)}$$

$$= \frac{K_1 s^2 + K_2 s + 7K_1 s + 7K_2 + (-35/53)s^2 + (\frac{-140}{53})}{(s+7)(s^2+4)}$$

so $K_1 - 35/53 = 0$

$K_2 + 7K_1 = 5$

$7K_2 - 140/53 = 0$

$K_1 = 35/53$

$K_2 = 20/53$

and $$F(s) = \frac{35/53\, s + 20/53}{s^2+4} + \frac{(-35/53)}{s+7}$$

$$= \left(\frac{35}{53}\right)\frac{s}{s^2+4} + \left(\frac{10}{53}\right)\frac{2}{s^2+4} + \left(\frac{-35}{53}\right)\frac{1}{s+7}$$

and $$f(t) = \left(\frac{35}{53}\right)\cos 2t\, u(t) + \left(\frac{10}{53}\right)\sin 2t\, u(t) + \left(\frac{-35}{53}\right)e^{-7t}u(t)$$

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or $$X(s) = \frac{8/21}{s} + \frac{8/28}{s+7} + \frac{(-8/12)}{s+3}$$

STEP 3 INVERSE LAPLACE transform

$$\mathcal{L}^{-1}\left[\frac{1}{s}\right] = u(t)$$
$$\mathcal{L}^{-1}\left[\frac{1}{s+a}\right] = e^{-at}u(t)$$

From Tables

so

$$x(t) = 8/21\, u(t) + 8/28\, e^{-7t}u(t) + -8/12\, e^{-3t}u(t)$$
$$= \frac{8}{21}u(t) + \frac{2}{7}e^{-7t}u(t) - \frac{2}{3}e^{-3t}u(t)$$