8-18

$$G(s)H(s) = \frac{K(s-1)(s-2)}{s(s+1)}$$

OL poles at 0, -1
OL zeros at +1, +2

Rule 1 – Since there are 2 poles and 2 zeros there are 2 branches.

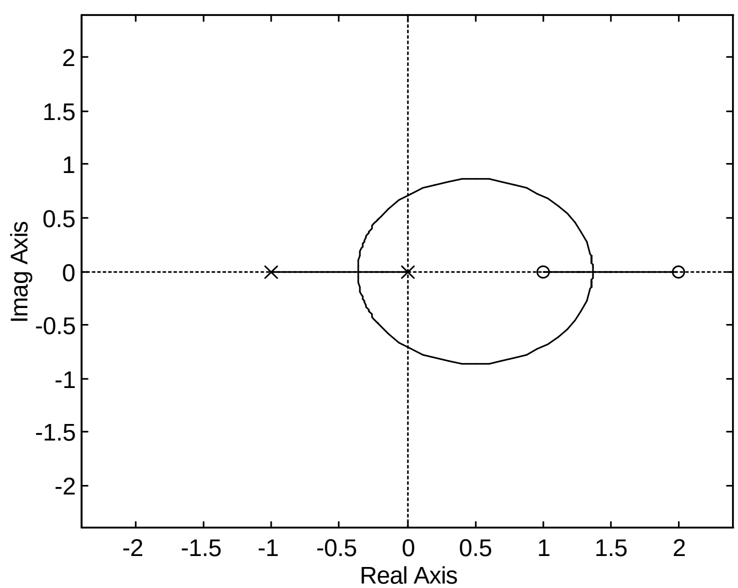
Rule 2 – The locus is symmetric about the real axis

Rule 3 – The locus begins at the poles (0,-1) and ends at the zeros (+1,+2)

Rule 4 – The root locus exists on the real axis when there are an odd number of poles, zeros to the right. In particular, the locus is on the axis between 1 & 2 and between 0 & -1

Rule 5 – Doesn't apply.

Here is our sketch:



a) Breakaway-Breakin points

Using rule 6:

$$\frac{-d}{ds}\left(\frac{1}{G(s)H(s)}\right)=\frac{-d}{ds}\left(\frac{s^2+s}{s^2-3s+2}\right)=$$

$$(s^2+s)*\frac{-d}{ds}\left(\frac{1}{s^2-3s+2}\right)+\left(\frac{1}{s^2-3s+2}\right)*\frac{-d}{ds}(s^2+s)=$$

$$(s^2+s)*(-1)\left(\frac{1}{s^2-3s+2}\right)^2(-1)(2s-3)+\left(\frac{1}{s^2-3s+2}\right)*(-2s-1)$$

$$\left(\frac{1}{s^2-3s+2}\right)^2(2s-3)(s^2+s)+(s^2-3s+2)(-2s-1)=$$

$$\left(\frac{1}{s^2-3s+2}\right)^2(4s^2-4s-2)$$

$$\text{and s}=-0.3660,\ 1.3660$$

Rule 6' instead

$$\frac{1}{s}+\frac{1}{s+1}=\frac{1}{s-1}+\frac{1}{s-2}$$

$$\frac{2s+1}{s(s+1)}=\frac{2s-3}{(s-1)(s-2)}$$

$$(2s+1)(s-1)(s-2)=s(s+1)(2s-3)$$

$$2s^3-5s^2+s+2=2s^3-s^2-3s$$

$$-4s^2-4s+2=0, and$$

$$s=-0.3660{,}1.3660$$

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b) jw axis crossings

Rule 7: Solve the characteristic equation 1+KGH=0 with s=jw

1+K(s-1)(s-2)/(s)(s+1)=0

s(s+1)=-K(s-1)(s-2)

s^2+s = -K (s^2-3s+2)

substitution s=jw,

-w^2 + jw = -K (-w^2 – 3jw + 2)

-w^2 + jw = j(3Kw) + (K w^2 – 2K)

equating real and imaginary on both sides

3Kw=w, or K=1/3

-w^2=K w^2 – 2K

0 = (4/3 w^2 – 2/3)

and w = sqrt(1/2) = +/- .7071

So the jw crossing happens when s= +/- .7071j and K=1/3

Notice that there is also a solution at w=0, K=0.

Rule 7' instead, Create the closed loop transfer function, G/(1+GH)

$$\frac{K(s-1)(s-2)}{s(s+1)+K(s-1)(s-2)} = \frac{K(s-1)(s-2)}{s^2(1+K)+s(1-3K)+2K}$$

s^2	1+K	2K
s^1	1-3K	0
s	2K	0

A row of all zeros when K=1/3 and when K=0.

And K=1/3, the denominator is s^2(4/3) + 2/3, or s=+/- .7071 j

So the jw crossing happens when s= +/- .7071j and K=1/3

There is also a solution at w=0, K=0 which is the open loop pole s=0.

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c) Since The locus starts at the poles when K=0 and the poles are in the LHP it is stable there. The locus crosses the jw axis at K=1/3 (see above) so it becomes unstable there.

   The system is stable 0<K<1/3

d) We want the damping ratio to be 0.5. Using sgrid and rlocfind in matlab, we find that the poles are at -.25 +/- .433j with K=.1429.

8-19

Since I gave gory detail for 8-18, I will simply give the answers here:

a) asymptotes: sigma=2.5 and theta=45, 135, 225, 315

b) Breakaway –1.38 and –3.62
c) jw crossing +/- 2.24j when K=126. Hence the system is stable 0<K<126
d) Using matlab, we search the .7 damping line and find poles are at -.992 +/- j 1.012 for K=10.32
e) The locus must now cross through the point j5.5 (read the paragraph above e). Therefore, the angle contributions of the poles and zeros must add up to an odd multiple of 180 degrees (this is the angle criterion, see your sheet or your notes).

   Before the zero is added, the poles and zeros have angles that add up to –265.074. Therefore the contribution of the zero must be 265/074-180=85.074. In order for this to happen, you must place the zero at .474.

f) After adding the zero, the locus crosses the imaginary axis at K=252.5
g) The new locus crosses the .7 damping ratio line farther away from the origin (wn is now bigger). Therefore it has a shorter settling time, and the OS is identical.