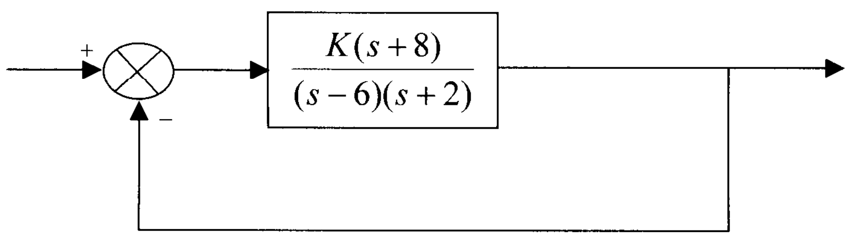
Name:

Honor Code: KEY

Instructions:

- Use the space on the accompanying pages to work the problems. Do not use a bluebook. Attach additional worksheets if necessary.
- If you wish to have partial credit awarded for any of your incorrect answers **you must write clearly and legibly.** Explain your work in words, if necessary.
- Read the instructions provided with each problem.
- Don't Panic.

1. [25] Draw the root locus for the following system



i) [1] List the finite poles, finite zeros, number of infinite zeros and number of infinite poles

fp: 6, -2
fz: -8
ip: NONE
iz: 1

ii) [2] Where does the locus lie on the real axis?

Between $(-2 \;\&\; 6)$ & $(-\infty \;\&\; -8)$

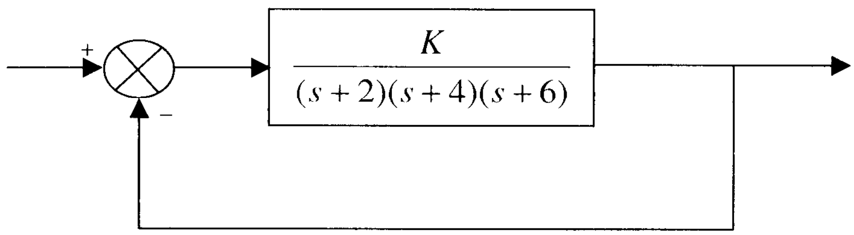
iii) [2] Find the asymptotes as $K \to \infty$ (if any). If there are none, explain why.

σ – θ –

$$\sigma = \frac{6-2-(-8)}{2-1} = \frac{12}{1} = \underline{12}$$

$$\theta = (2k+1)\pi = \underline{180^\circ}$$

2. [20] It is desired that the following system operate with 10% overshoot and less than 1s settling time. Currently, the system is operating with 10% overshoot and 1.88s settling time.



i) [3] The compensated system must have 10% overshoot and less than 1s settling time.

a) What is the zeta this requires?

b) What is the ***range*** of w_n this requires?

$\boxed{\zeta = .5901}$

$\frac{4}{\zeta w_n} < 1$ $\quad$ $\zeta w_n > 4$ $\quad$ $w_n > 4/.5901$ $\quad$ $\boxed{w_n > 6.77}$

ii) [17] Design a PD controller that exploits pole-zero cancellation to meet the specifications for the final system. Notice that pole-zero cancellation will turn this into a purely second order system. List

a) The location of your zero

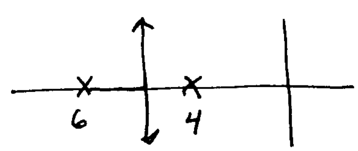
b) The gain your system requires

c) Draw the new root locus

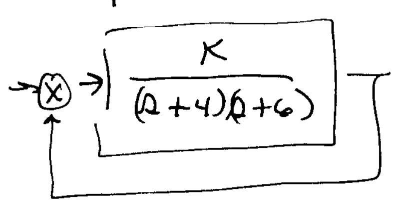
d) The overshoot and settling time of your new system.

You must write neatly and clearly if you want partial credit. Explain your work in words.

a) Choosing a zero at $s = 2$ will work

c) 

New root locus

b) The new system is 

$$CLTF = \frac{K}{s^2 + 10s + 24 + K}$$

We want to meet $\zeta = .5901$

and $\omega_n < 8$ (to safely meet specs)

$$s^2 + 10s + (24+K) = s^2 + 2\zeta\omega_n s + \omega_n^2$$

$$10 = 2\zeta\omega_n$$

$$\omega_n = 5/\zeta = 5/.5901 = \underline{8.47}$$

$$K = \omega_n^2 - 24 = \underline{47.5}$$

d) the overshoot is $.\cancel{1}0\%$ (as required)

$$T_s = 4/\zeta\omega_n = 4/5 = 0.8\ s$$

in spec.

3. [16] Circle the best answer.

i. (-3-2j) in exponential coordinates is

a. $3.6e^{33.69°}$

b. $3.6e^{0.5880°}$

c. $3.6e^{-146.31°}$

d. $3.6e^{3.6056°}$

ii. $\mathcal{L}\{(e^{3t} - e^{-7t})u(t)\} =$

a. $\frac{1}{s-3} + \frac{1}{s+7}$

b. $\frac{1}{s+3} + \frac{1}{s-7}$

c. $\frac{1}{s-3} - \frac{1}{s+7}$

d. $\frac{1}{s+3} - \frac{1}{s-7}$

iii. $\mathcal{L}^{-1}\left\{\frac{1}{s^2+6s+5}\right\} =$ $\frac{-1/4}{s+5} + \frac{+1/4}{s+1}$

a. $0.25(e^{-t} - e^{-5t})u(t)$

b. $0.25(e^{-t} + e^{-5t})u(t)$

c. $0.25(e^{t} - e^{5t})u(t)$

d. $0.25(e^{t} + e^{5t})u(t)$

iv. A system with peak time 1s and settling time 2s has

a. zeta=0.4553 and w_n=1.0825

b. zeta=0.0319 and w_n=19.74

c. zeta=0.5370 and w_n=3.7242

d. zeta=0.7484 and w_n=2.6724

$T_s = \frac{4}{\zeta\omega_n}$ $2 = \frac{4}{\zeta\omega_n}$ $\zeta\omega_n = 2$

$\omega_n^2 - \omega_n^2\zeta^2 = \pi^2$

$T_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}}$

$\omega_n^2 = \pi^2 + 4$

$\omega_n = \sqrt{\pi^2+4}$

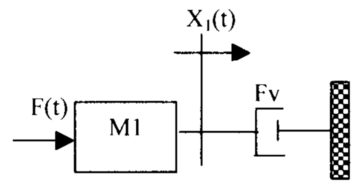
$\omega_n\sqrt{1-\zeta^2} = \pi$

$\omega_n^2(1-\zeta^2) = \pi^2$

v. $\frac{X_1(s)}{F(s)}$ is a second order system

a. True

b. False



vi. A unity feedback system with $G(s)=(s-2)/(s^2+3s+1)$ as the open loop transfer function is stable.

a. True

b. False

vii. You wish to design a controller that reduces the steady state error and reduces the overshoot. You could use

a. Lead Controller

b. Lag Controller

c. PD Controller

d. Lead/Lag Controller

viii. A row of all zeros in a Routh Table means

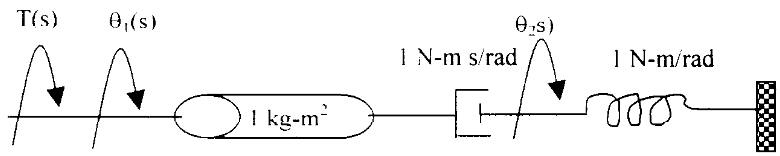
a. The system is unstable

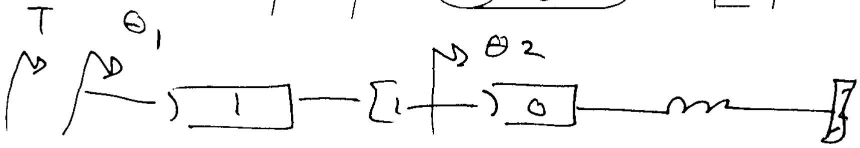
b. The poles of the system are symmetric about the origin

c. There is at least one pole on the jw axis.

d. All of the above

4. [14 points] Find the settling time and overshoot for $\dfrac{\theta_2(s)}{T(s)}$ when T(t) is a unit step.





$$T(s) = \theta_1(s)(s^2+s) + \theta_2(s)(-s)$$

$$0 = \theta_1(s)(-s) + \theta_2(s)(s+1)$$

$$\theta_1(s) = \theta_2(s)\left(\frac{s+1}{s}\right)$$

$$T(s) = \left[s^2+s\cdot\left(\frac{s+1}{s}\right)\right]\theta_2(s) + \theta_2(s)(-s)$$

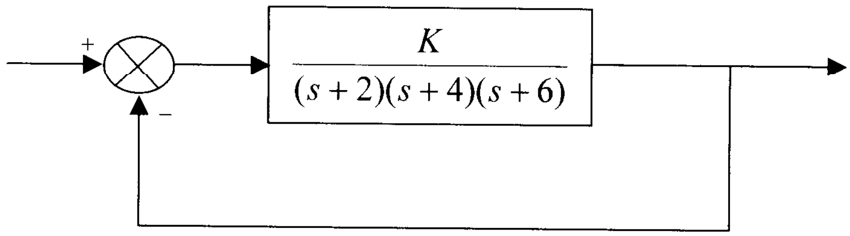
$$\theta_2(s)\left[s^2+s+1\right] = T(s)$$

$$\frac{\theta_2(s)}{T(s)} = \frac{1}{s^2+s+1} \qquad \omega_n = 1$$

$$\zeta = 1/2$$

$$\boxed{\begin{array}{l} T_s = 8s \\ OS = 16.3\% \end{array}}$$

5. [10] We wish to design a lag controller to reduce the steady state error in the following system by a factor of 10. The system currently operates with K=45.9.



i) [2] What is the position error constant and corresponding steady state error due to a step input in the uncompensated system?

$$\lim_{s\to 0} \frac{1}{1+G(s)} = \frac{1}{1+\frac{45.9}{2\cdot 4\cdot 6}} = \frac{1}{1+\frac{45.9}{48}} = \underline{.511}$$

$$K_p = \frac{45.9}{48} \simeq .956$$

ii) [3] Choose an appropriate lag compensator to reduce the SSE by a factor of 10.

~~Kp~~ - we'd like SSE = .0511

we'd like $K_p = 18.61 = 19.47 \times .956$

Choose $\dfrac{s+.1947}{s+.01}$

iii) [2] Comment on whether your controller is physically realizable or not and why.

Buildable since pole/zero location is governed by 1/RC.

iv) [3] When K = 45.9 (the current operating point of the system), it is found the closed loop poles are at –7.9, -2.04+2.72j and –2.04-2.72j. Comment on how your compensator would effect the transient response and why.

The Pole & zero are close to the origin (a factor of 10 in from dominant poles) → very little affect on transient.

6. [15] Sketch the Bode Magnitude and Phase plots for the following.

$$G(s) = \frac{10(s+2)}{(s+1)(s+3)(s+4)}$$

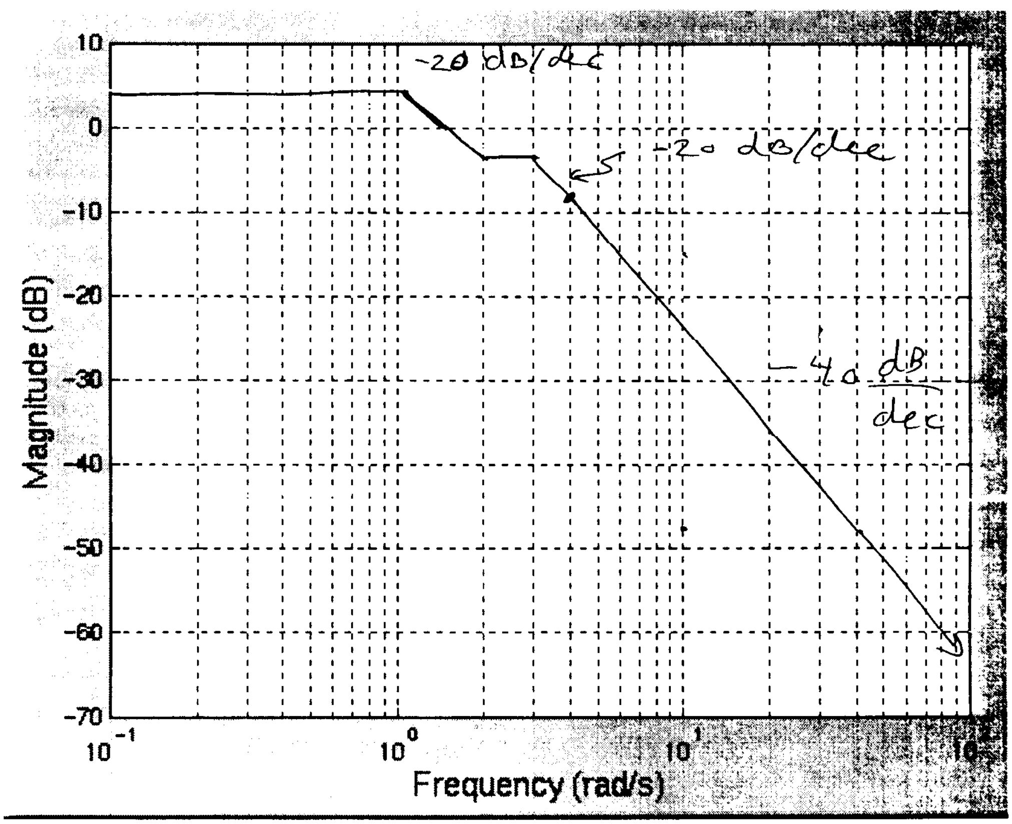
a. [7] Sketch the Bode Magnitude plot asymptotically.

$$G(s) = \left(\frac{5}{3}\right) \frac{\frac{s}{2}+1}{(s+1)\left(\frac{s}{3}+1\right)\left(\frac{s}{4}+1\right)}$$

Starts at $20 \log_{10} (5/3) = 4.44 \text{ dB}$

0 initial slope

break freqs @ $\omega = 1, 2, 3, 4$



b. [8] Sketch the Bode Phase plot.

i. Find the low frequency asymptote

ii. Find the high frequency asymptote

iii. Find the exact phase at the break frequencies

i at $\omega = 0$, $-90 \times 0 = 0^\circ$

ii at $\omega = \infty$, $-90(3-1) = -180^\circ$

iii ~~For~~ $\angle G(j\omega) = \angle(s+2) - \angle(s+1) - \angle(s+3) - \angle(s+4)$

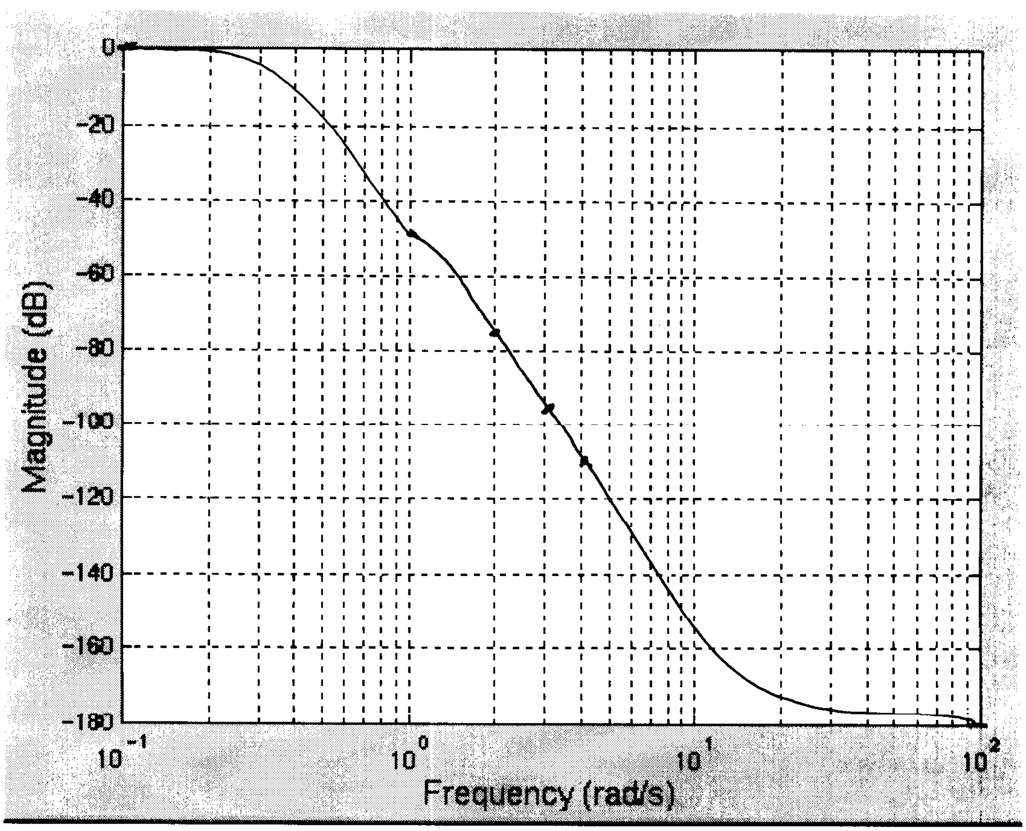
$= \tan^{-1}(\omega/2) - \tan^{-1}(\omega) - \tan^{-1}(\omega/3) - \tan^{-1}(\omega/4)$

$\omega = 1 \quad -50.91^\circ$

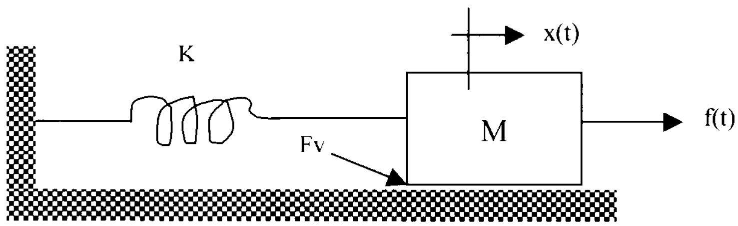
$\omega = 2 \quad -78.69^\circ$

$\omega = 3 \quad -97.13^\circ$

$\omega = 4 \quad -110.66^\circ$



4. [~~10~~ 14 points] Solve for x(t) in the following system if f(t) is a unit step. You may use M=1 kg, K=5 N/m, Fv=1 N-s/m.



$$F(s) = (Ms^2 + F_v s + K)\,X(s)$$

$$F(s) = (s^2 + s + 5)\,X(s)$$

$$\frac{X(s)}{F(s)} = \frac{1}{s^2+s+5} \qquad X(s) = \frac{1}{s(s^2+s+5)}$$

$$x(t) = \mathcal{L}^{-1}[X(s)] \qquad X(s) = \frac{A}{s} + \frac{Bs + C}{s^2+s+5}$$

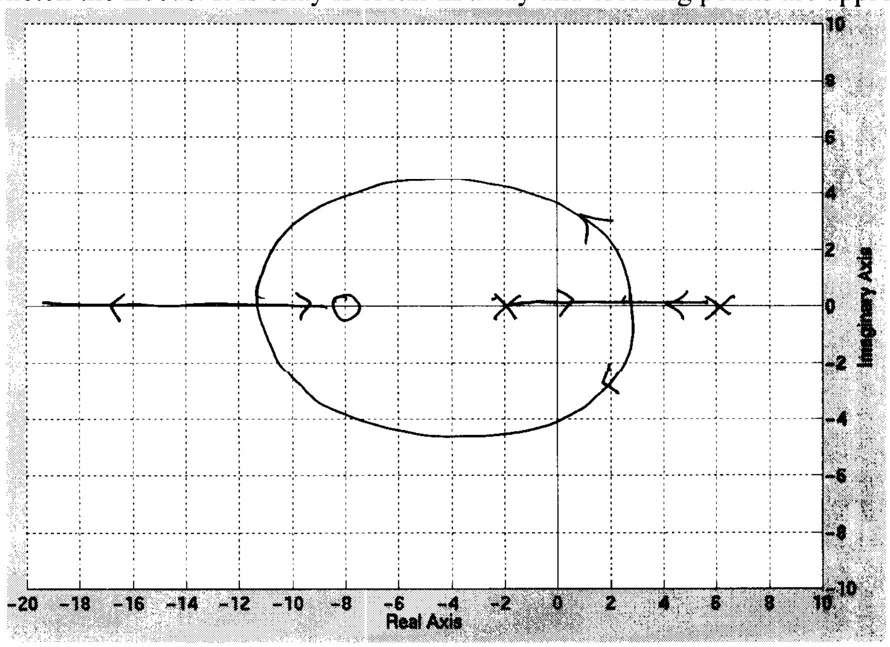
$$= \frac{0.2}{s} + \frac{-0.2s - 0.2}{s^2+s+5}$$

$$= \frac{0.2}{s} + \frac{-0.2s - 0.2}{(s+1/2)^2 + \sqrt{4.75}^2}$$

$$= \frac{0.2}{s} + \frac{-0.2(s+1/2) + \frac{-0.2}{\sqrt{4.75}}\sqrt{4.75}}{(s+1/2)^2 + \sqrt{4.75}^2}$$

$$x(t) = 0.2\left[1 - e^{-\frac{1}{2}t}\cos\sqrt{4.75}t - \frac{e^{-\frac{1}{2}t}\sin\sqrt{4.75}\,t}{\sqrt{4.75}}\right] u(t)$$

iv) [5] Sketch the Locus. It is okay if break-in/away and crossing points are approximate.



v) [4] Find the breakaway and break-in points if any. If there are none explain why.

$$\frac{1}{s+8} = \frac{1}{s-6} + \frac{1}{s+2}$$

$$\frac{1}{s+8} = \frac{2s-4}{(s-6)(s+2)}$$

$$s^2 - 4s - 12 = 2s^2 - 4s + 16s - 32$$

$$0 = s^2 + 16s + 20$$

$$s = \frac{-16 \pm \sqrt{256 - 4(-20)}}{2} = -8 \pm \frac{\sqrt{336}}{2}$$

$$-8 \pm \frac{\sqrt{18.4}}{2}$$

$$= \underline{1.2} \text{ and } \underline{-17.2}$$

vi) [5] Find the value of K and s at the ***three*** jw axis crossings.

R-H table on $\dfrac{K(s+8)}{s^2-4s-12+K(s+8)}$

denominator $= s^2 + s(K-4) + (8K-12)$

s^2	1	$8K-12$
s^1	$K-4$	
s^0	$8K-12$	

has RoZ @ $K=4$
$K=1.5$

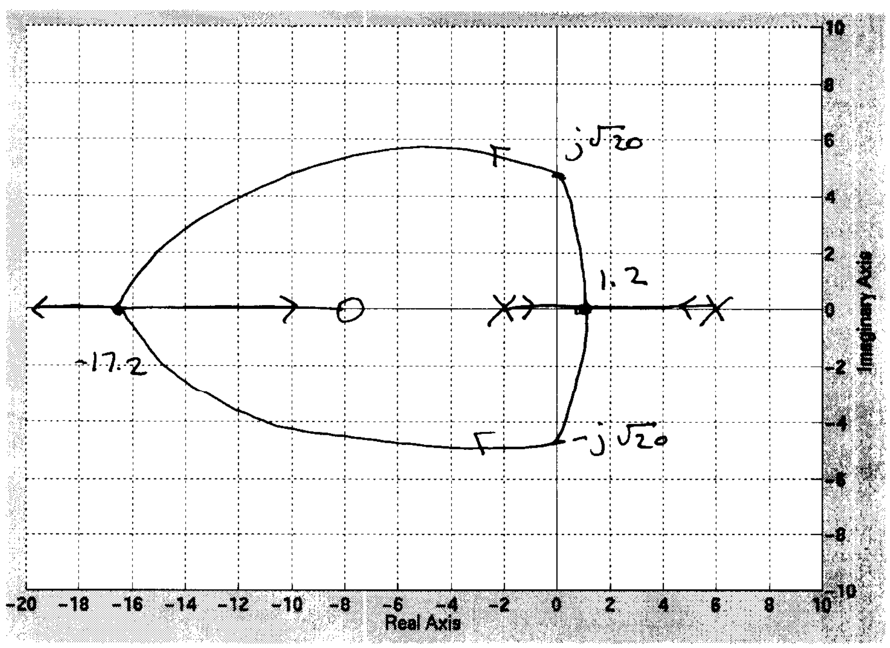
K	s
4	$j\sqrt{20}$
4	$-j\sqrt{20}$
1.5	0

at $K=4$, $s^2+20=0$ $\quad s = \underline{\pm j\sqrt{20}}$

at $K=1.5$ $\quad s^2+(-2.5s)=0$ $\quad \underline{s=0}$ or 2.5

only s with 0 real part are valid

vii) [4] Make a complete drawing of the locus incorporating v and vi. Label everything (break-in, breakaway points and both jw axis crossings)



viii) [2] What is the range of K for stability?

$\underline{K > 4}$