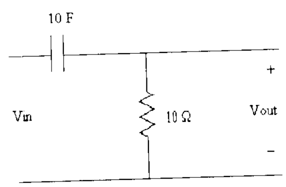
5. [20] Answer the following questions.

i. [T F] A second order system that is underdamped has two roots on the real axis.

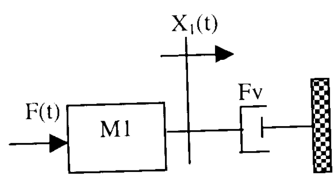
ii. [T F] The closed loop transfer function $T(s) = \dfrac{1}{(s^2+2s+10)(s+9)}$ can be approximated as second order, based on the standard rule of thumb.

iii. [T F] The RC circuit shown below has

$$\frac{Vout(s)}{Vin(s)} = \frac{1}{s+1}$$



iv. [T F] $\dfrac{X_1(s)}{F(s)}$ is a second order system



v. [T F] The equation $Ts = 4/\zeta\omega_n$ is an approximation.

vi. [T F] A system with ζ near 1 will have more oscillations than one with ζ near 0.

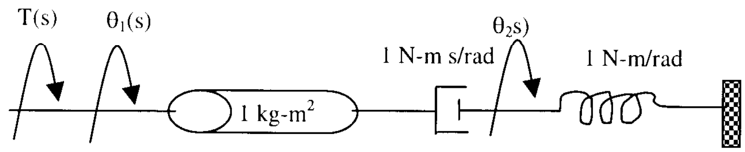
vii. [T F] A system with closed loop denominator $s^2 - 2s + 3$ is stable.

viii. [T F] A system can have both 0 SSE due to a step input and 0 SSE due to a ramp input.

ix. [T F] The root locus is a plot of the position of a systems closed loop poles as the gain is varied.

x. [T F] Bode plots contain stability information.

4. [15 points] Find the settling time and overshoot for $\frac{\theta_2(s)}{T(s)}$ when T(t) is a unit step.



eq 1: $T(s) = s^2\theta_1(s) + s(\theta_1(s) - \theta_2(s))$

$T(s) = \theta_1(s)[s^2+s] + \theta_2(s)[-s]$

eq 2: $0 = s(\theta_2(s) - \theta_1(s)) + \theta_2(s)$

$\theta_1(s) = \theta_2(s)(s+1)/s$

$\theta_2(s) = \frac{s}{s+2}\theta_1(s)$

$\Rightarrow \quad T(s) = \left[\theta_2(s)\frac{(s+1)}{s}\right](s^2+s) - s\theta_2(s)$

$= \theta_2(s)\left[(s+1)^2 - s\right]$

$= \theta_2(s)\left[s^2 + \cancel{2}s + 1\right]$

$\frac{\theta_2(s)}{T(s)} = \frac{1}{s^2+s+1}$

$\omega_n = 1$

$\zeta = .5$

$\Rightarrow T_s = 8\ s$

$OS = 16.3\%$

2. [25] A unity feedback control system has

$$G(s) = \frac{K}{s(s+2)(s+10)}$$

a. [18] Design a compensator to yield dominant poles with a damping ratio of 0.357 and natural frequency of 1.6 rad/s.
b. [3] Estimate the expected overshoot and settling time.
c. [2] Is your second-order approximation valid?
d. [2] Justify the type of controller you choose.

a) $\zeta = 0.357$, $\omega_n = 1.6$ $\Rightarrow$ desired pole @ $.5712 \pm j\,1.49$

Choose a lead and cancel $(s+10)$

$$\boxed{\frac{K(s+10)}{s+p}}$$

$\measuredangle$ from $s=0$ to $.5712 + j1.49$ is $110.97°$

$\measuredangle$ From $s=2$ to $.5712 + j1.49$ is $46.20°$

$157.17°$

$\measuredangle$ from p must be $22.82°$

Place it at $3.54 + .5712 = \underline{4.11}$

Find $K = \prod |s-p|$

$= |.5712 + j1.49|\,|.5712 + j1.49 + 4.11|\,|.5712 + j1.49 + 2|$

$= 1.596 \cdot 4.913 \cdot 2.972$

$= \underline{23.30}$

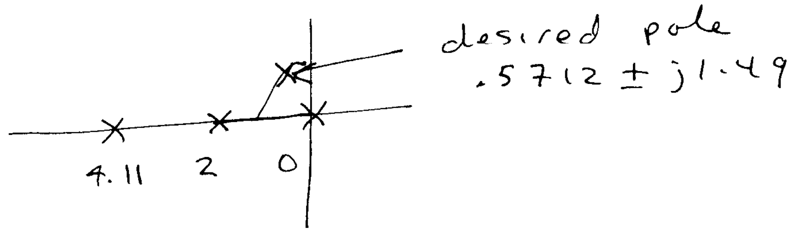
Final Design:

$$\boxed{\frac{23.3\,(s+10)}{s+2}}$$

b) $T_s \approx 4/(.357 \cdot 1.6) = \underline{7\ s}$

$OS = \exp^{-\zeta\pi/\sqrt{1-\zeta^2}} = \underline{30.11\ \%}$

c) <u>Yes</u>



open loop pole at 4.11 moves left, so it is at least 4.11 units away from the origin, and this is more than 5 X .5712.

d) A Lead Controller is necessary to alter transient response. Simple PD Control won't work.

3. [15] Plot the asymptotic Bode magnitude and phase plots for

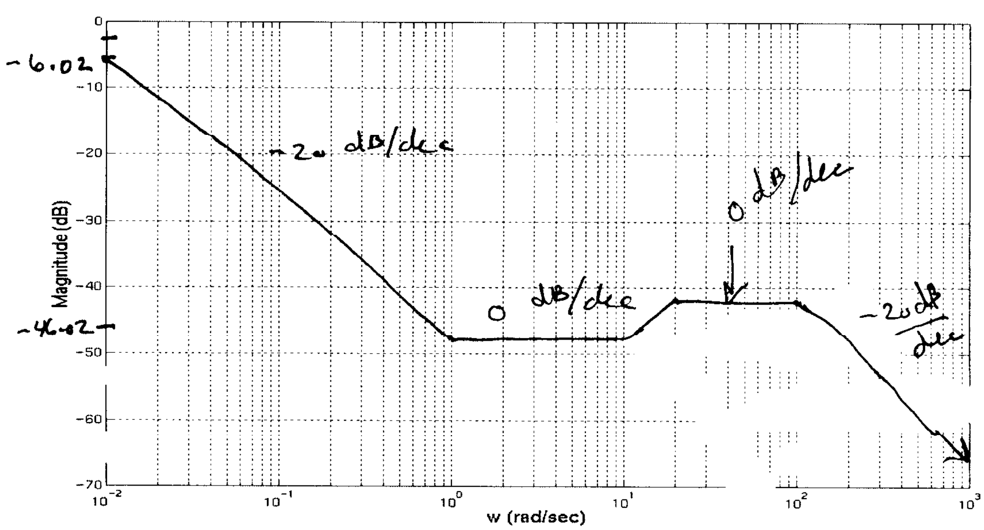
$$G(s) = \frac{(s+1)(s+10)}{s(s+100)(s+20)}$$

a. [9] Magnitude

$$G(s) = \frac{\left(\frac{s}{1}+1\right)\left(\frac{s}{10}+1\right)\cdot 10}{s\left(\frac{s}{100}+1\right)\left(\frac{s}{20}+1\right)\cdot 20\cdot 100}$$

- Starts at $+20\log_{10}\left(\frac{10/2000}{.01^{1}}\right) = -6.02\,dB$
- Initial slope $= -20$ dB/dec

Break freqs at 1, 10, 20, 100



b. [6] Phase. Calculate the exact phase at each of the break freqencies.

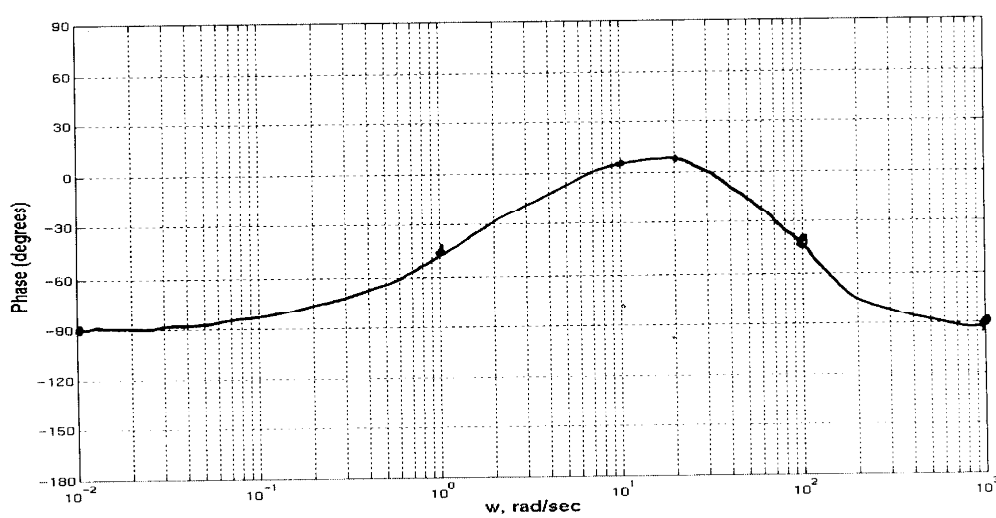
$$G(j\omega) = \frac{(j\omega+1)(j\omega+10)}{(j\omega)(j\omega+100)(j\omega+20)}$$

$$\angle G(j\omega) = \operatorname{atan}(\omega) + \operatorname{atan}(\omega/10) - 90 - \operatorname{atan}(\omega/100) - \operatorname{atan}(\omega/20)$$

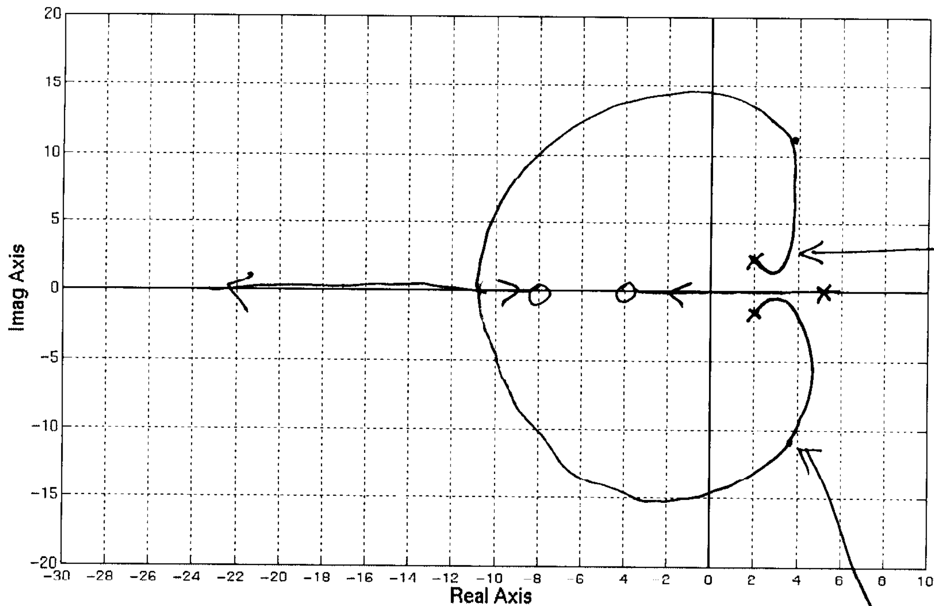
Freq	Phase
1	$-42.72°$
10	$7.01°$
20	$4.26°$
100	$-40°$

Starts at $-90°$

ends at $-90°$



viii) [3] Make a complete drawing of the locus incorporating vi and vii.



∠ of departure $-28.57°$

Passes thru $3.875 \pm j10.58$

vi) [3] Find the location of all of the closed loop poles when K = 1.25.

at $K = 1.25$, CL denominator is

$s^3 + s^2(-7.75) + s(43) + 0$

$= s(s^2 - 7.75s + 43)$

poles at $s = 0$

$s = 3.875 \pm 10.58j$

vii) [4] Find the angle of departure from the two complex poles.

angle of departure from $2+2j$:

$\measuredangle$ from $2-2j = 90^\circ$ (pole)

$\measuredangle$ from $5 = 180 - \tan^{-1}(2/3) = 146.31^\circ$

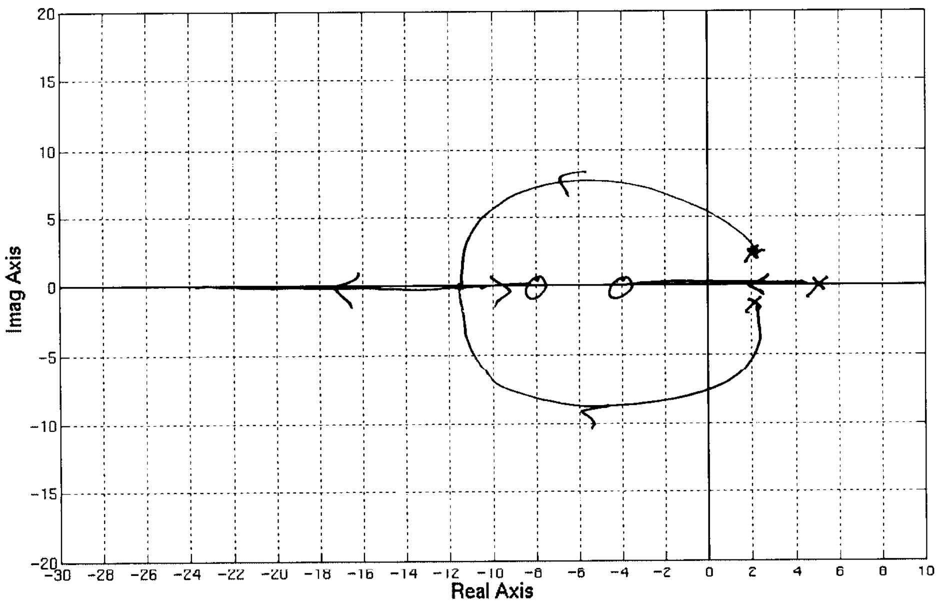
$\measuredangle$ from $-4 = \tan^{-1}(2/6) = 18.43^\circ$

$\measuredangle$ from $-8 = \tan^{-1}(2/10) = 11.31^\circ$

$\sum_i \measuredangle$ poles $- \sum_i \measuredangle$'s zeros $= 206.57^\circ$

$\therefore$ $\measuredangle$ of departure must be $\underline{-26.57^\circ}$

iv) [5] Sketch the Locus. It is okay if break-in/away and crossing points are approximate.



v) [5] Find the value of K at the ***three*** jw axis crossings.

Simplify to CLTF

$$\frac{K(s+4)(s+8)}{(s-5)(s^2-4s+8)+K(s+4)(s+8)}$$

$$\frac{K(s+4)(s+8)}{s^3+s^2(K-9)+s(28+12K)+(-40+32K)}$$

s^3	1	$28+12K$
s^2	$K-9$	$-40+32K$
s^1	$\dfrac{12K^2-112K-212}{K-9}$	0
s^0	$-40+32K$	0

$\Rightarrow K = 4.667 \pm 6.28 = \begin{cases} 10.94 \\ \cancel{-1.61} \end{cases}$

$\Rightarrow K = 1.25$

ECE 365 Final Exam S/S 2000

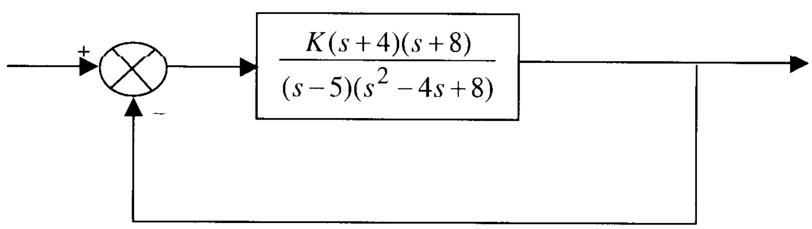
Name:

Honor Code: KEY

Instructions:

- Use the space on the accompanying pages to work the problems. Do not use a bluebook. Attach additional worksheets if necessary.
- If you wish to have partial credit awarded for any of your incorrect answers **you must write clearly and legibly.** Explain your work in words, if necessary.
- Read the instructions provided with each problem.
- Don't Panic.

1. [25] Draw the root locus for the following system



i) [1] List the finite poles, finite zeros, number of infinite zeros and number of infinite poles

fp: $+5$, $+2 \pm j2$

fz: -4, -8

ip: 0

iz: 1

ii) [2] Where does the locus lie on the real axis?

between $(+5$ and $-4)$ and $(-8$ and $-\infty)$

iii) [2] Find the asymptotes as $K \to \infty$ (if any). If there are none, explain why.

$\sigma =$ $\theta =$

$$\sigma = \frac{5 + (2+j2) + (2-j2) - (-4-8)}{3-2} = \underline{21}$$

$$\theta = \frac{180}{1} = \underline{180^\circ}$$