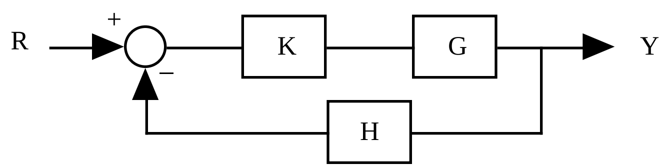
Rules for Constructing Root Locus for $0 \le K \le \infty$:



$$\text{Loop Transfer Function: } KGH = K\frac{G_N}{G_D}\frac{H_N}{H_D} = K\frac{\prod_{i=1}^{n_z}(s-z_i)}{\prod_{i=1}^{n_p}(s-p_i)}$$

$$\text{Closed -loop Transfer Function: } \frac{KG}{1+KGH} = \frac{\frac{KG_N}{G_D}}{1+\frac{KG_N H_N}{G_D H_D}} = \frac{KG_N H_D}{G_D H_D + KG_N H_N}$$

Characteristic Equation : $1 + KGH = 0$ OR $G_D H_D + KG_N H_N = 0$

$$K = -\frac{1}{GH} = -\frac{G_D H_D}{G_N H_N} = -\frac{\prod_{i=1}^{n_p}(s-p_i)}{\prod_{i=1}^{n_z}(s-z_i)}$$

$$\sum_{i=1}^{n_p}\text{angles}\{(s-p_i)\} - \sum_{i=1}^{n_z}\text{angles}\{(s-z_i)\} = \pm k(180^o),\ \ k - 1,3,5,\ldots \quad \text{Angle Criterion}$$

and $$K = \frac{\prod_{i=1}^{n_p}|(s-p_i)|}{\prod_{i=1}^{n_z}|(s-z_i)|}$$ Magnitude Criterion

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Rule 1. The root locus has $n = \max(n_p, n_z)$ branches where n_p is the number of finite poles of the loop transfer function and n_z is the number of finite zeros of the loop transfer function.

Rule 2. The root locus is symmetric about the real axis in the s-plane.

Rule 3. The root locus branches begin ($K = 0$) at the loop transfer function poles and end ($K = \infty$) on the loop transfer function zeros.

Rule 4. The root locus exists on the real axis if there is an odd number of poles and zeros (of the loop transfer function) to the right on the real axis.

Rule 5. The branches of the root locus which go off to the loop transfer function zeros at infinity approach asymptotically the straight lines with angles

Angles of the asymptotes $: \theta = \frac{\pm k(180^o)}{n_p - n_z}, \quad k = 1,3,5,...$

These asymptotes intersect the real axis at

$$\sigma = \frac{\sum_{i=1}^{n_p} p_i - \sum_{i=1}^{n_z} z_i}{n_p - n_z}$$

where p_i and z_i are respectively the finite poles and zeros of the loop transfer function. The number of asymptotes is equal to $(n_p - n_z)$

Rule 6. The root locus branches intersect the real axis at points where $K = \frac{-1}{G(s)H(s)}$ is at an extremum for real values of s, i.e., for real values of s where $\frac{d}{ds}\left(\frac{-1}{G(s)H(s)}\right) = 0$.

Rule 7. $j\omega - axis$ crossings may be determined by solving the characteristic equation with $s = j\omega$.

Rule 8 (a) The root locus angle of departure from a complex pole p_c can be found by subtracting from 180^0 the angle contribution at this pole of all other finite poles and adding the angle contribution of all finite zeros of the loop transfer function.

$$\theta_d\Big|_{s=p_c} = 180^0 - \sum(\text{Angles of all other finite poles to } p_c) + \sum(\text{Angles of all finite zeros to } p_c)$$

Rule 8 (b) The root locus angle of arrival at a complex zero z_c can be found by adding to (-180^0) the angle contribution at this zero of all finite poles and subtracting the angle contribution of all other finite zeros of the loop transfer function.

$$\theta_a\Big|_{s=z_c} = -180^0 + \sum(\text{Angles of all finite poles to } z_c) - \sum(\text{Angles of all other finite zeros to } z_c)$$

Rule 9. The value of K corresponding to a point $s = s_r$ on the root locus is determined by using the magnitude criterion:

$$K = \frac{\prod_{i=1}^{n_p} |(s_r - p_i)|}{\prod_{i=1}^{n_z} |(s_r - z_i)|}$$

In case of systems with positive feedback instead of negative feedback:

$$\text{Closed -loop Transfer Function: } \frac{KG}{1-KGH} = \frac{\frac{KG_N}{G_D}}{1-\frac{KG_N H_N}{G_D H_D}} = \frac{KG_N H_D}{G_D H_D - KG_N H_N}$$

In *Rule 4* , replace *odd* by ***even***..

In *Rule 5*, change θ to: $\theta = \frac{\pm k(360^o)}{n_p - n_z}, \quad k = 1,2,3,...$

In *Rule 8*, change 180^o to **360º**.