ECE 273 Midterm Exam

27 February 2002
Version C

Name: KEY

Honor Code:

- All Exams are due promptly at 6:00 PM.
- If you have a question, **ask it** by coming to my desk. All questions and answers of consequence will be repeated aloud to the class.
- There are five problems: Working them in order is not necessarily the best strategy.
- Clearly circle or box all of your final answers. If I can't find your answer, it's wrong.

[20 Points] 1. Simplify each of the following expressions to minimum SOP form.

(a) $f(a,b,c) = ab + abc + abc' + ac$

(b) $(a'+b)(a'+b'+c)(b+d+e)(a+b'+e)$

(c) $f(a,b,c,d) = adb + bda + a'c'$

(d) $f(a,b,c,d,e) = (a'+b+c'+e')(b+c'+d+e')(a+b+c'+e')$

(e) $f(a,b,c) = \left(\sum m(0,1,2,3)\right)\left(\prod M(1,3,7)\right)$

(a) $ab + abc + abc' + ac = \boxed{ab + ac}$

(b) $\underbrace{(a'+b)(a'+b'+c)}(b+d+e)(a+b'+e)$ use $(x+y)(x+z) = x+yz$

$= \left(a' + b(b'+c)\right)\left(e + (b+d)(a+b')\right)$

$= (a' + bc)(e + ab + ad + b'd)$

$= a'e + a'b'd + bce + abc + \cancel{abcd}$ (bce consensus)

$= \boxed{a'e + a'b'd + abc}$

(c) $adb + bda + a'c' = \boxed{abd + a'c'}$

(d) $(a'+b+c'+e')(b+c'+d+e')(a+b+c'+e')$

let $x = c' + b + e'$

$(x + a')(x + d)(x + a) = (x + a'd)(x + a)$

$= x = \boxed{c' + b + e'}$

(e) $\left(\sum m(0,1,2,3)\right)\left(\prod M(1,3,7)\right) = \sum m(0,1,2,3) \sum m(0,2,4,5,6)$

$= \sum m(0,2)$

$= a'b'c' + a'bc'$

$= \boxed{a'c'}$

[16 Points] 2. Answer True or False to the following statements.

T (a) $abc + ab'c' + b'cd + bc'd + ad = abc + ab'c' + b'cd + bc'd$

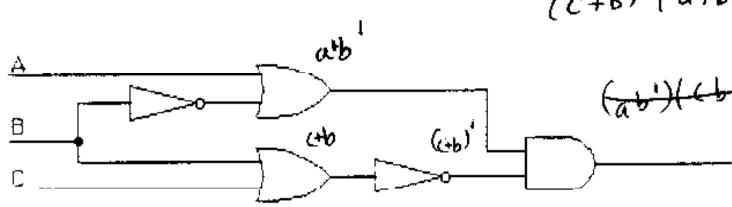
F (b) $(x+y)(y+z)(x+z) = (x'+y')(y'+z')(x'+z')$

F (c) Given $f_1(a,b,c) = \sum m(0,1,2,3,4,7)$ and $f_2(a,b,c) = \sum m(0,1,2,3,5,6)$ then $f_1 = f_2$

F (d) The circuit at below is equivalent to the logic expression $b'+c'a$

$(c+b)'(a+b') = c'b'(a+b')$
$= ac'b' + c'b' = c'b'$

~~$(a+b')(c+b)' = ab'(c'+b')$~~
~~$= ab'c' + ab'$~~
~~$= ab'$~~



(a) Need to show ad is redundant; to be true or a counter example if false

LHS

cd \ ab	00	01	11	10
00				1
01		1	1	1
11	1		1	1
10			1	

RHS

cd \ ab	00	01	11	10
00				1
01		1	1	1
11	1		1	1
10			1	

Kmaps the same

alternatively:
Consensus $abc, b'cd \rightarrow acd$
$ab'c', bc'd \rightarrow ac'd$
Consensus $acd, ac'd \rightarrow ad$

(b) $(x+y)(y+z)(x+z) = (y+xz)(x+z) = xy + xz + yz$
$(x'+y')(y'+z')(x'+z') = \ldots \quad x'y' + x'z' + y'z'$
Clearly different (let $x = y = z = 1$)

(c) This would imply some minterms are redundant; not possible in a minterm expansion

(d) See Figure

[20 Points] 3. Complete the following short problems.

(a) Write $wx'y + wxz' + y'z$ in minimal POS form.

(b) Given $f(a,b,c,d) = \prod M(0,1,2,3,7,9,15)$, write f' in minterm form.

(c) Given $f(a,b,c,d) = \prod M(0,1,2,3,7,9,15)$, write f in minimum POS form.

(d) Given $f(a,b,c,d) = \prod M(0,1,2,3,7,9,15)$, write f in minimum SOP form.

(a)

yz \ wx	00	01	11	10
00	0	0	1	0
01	1	1	1	1
11	0	0	0	1
10	0	0	1	1

$F' = w'y + w'z' + xyz + x'y'z'$

$(F')' = (w+y')(w+z)(x'+y'+z')(x+y+z)$

(b) $f = \sum m(4,5,6,8,10,11 \dots 14)$

$f' = \sum m(0,1,2,3,7,9,15)$

(c) $f = \sum m(4,5,6,8,10,11,12,13,14)$

$F' = a'b' + b'c'd + bcd$

$F = (a+b)(b+c+d')(b'+c'+d')$

cd \ ab	00	01	11	10
00	0	1	1	1
01	0	1	1	0
11	0	0	0	1
10	0	1	1	1

(d) Minimum SOP found from Kmap above

$f = bc' + bd' + ab'c + ad'$

[24 Points] 4. You are to design a **4 bit counter**. The four bits of the counter are to be read as a single binary number, e.g.

if $ABCD = 0010$, the counter state is 2 (since $0010_{(2)} = 2_{(10)}$)

The next state of the counter $(A^+B^+C^+D^+)$ is to be 2 times the current state. If the resulting next state is greater than 15, the next state is to be 2 times the current state minus 15.

[7] (a) Draw a truth table showing all current state $(ABCD)$ and next state $(A^+B^+C^+D^+)$ combinations.

[13] (b) Determine the inputs (J_a, K_a, J_b, … K_d) if the counter is to implemented using JK Flip-flops. Note that the state/next state relationship for a JK flipflop is $Q^+ = JQ' + K'Q$.

[4] (c) Draw a circuit diagram for the counter.

(a)

abcd	$a^+b^+c^+d^+$
0000	0000
0001	0010
0010	0100
0011	0110
0100	1000
0101	1010
0110	1100
0111	1110
1000	0001
1001	0011
1010	0101
1011	0111
1100	1001
1101	1011
1110	1101
1111	1111

(b)

a^+

cd \ ab	00	01	11	10
00	0	1	1	0
01	0	1	1	0
11	0	1	1	0
10	0	1	1	0

b^+

cd \ ab	00	01	11	10
00	0	0	0	0
01	0	0	0	0
11	1	1	1	1
10	1	1	1	1

c^+

cd \ ab	00	01	11	10
00	0	0	0	0
01	1	1	1	1
11	1	1	1	1
10	0	0	0	0

d^+

cd \ ab	00	01	11	10
00	0	0	1	1
01	0	0	1	1
11	0	0	1	1
10	0	0	1	1

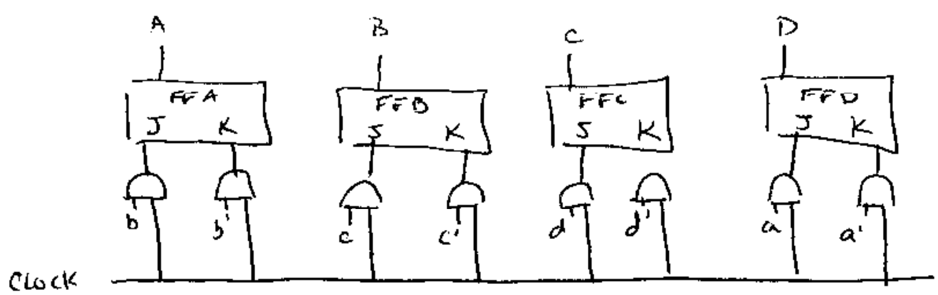
Short cut method

$J_a = b$ $\quad J_B = c$ $\quad J_c = d$ $\quad J_D = a$

$K_a' = b$ $\quad K_B' = c$ $\quad K_c' = d$ $\quad K_D' = a$

$(K_a = b')$ $\quad (K_B = c')$ $\quad (K_c = d')$ $\quad (K_D = a')$

(c)



[20 Points] 5. You are to **design a subtractor**. The subtractor has two inputs, X and Y. X has 3 bits and Y has 2 bits. The subtractor has one output Z, which has 3 bits.

The 3 bits of X represent a 3 bit binary number (e.g. if $X = 010$, then X represents the decimal number 2).

The 2 bits of Y represent a 2 bit binary number (e.g. if $Y = 11$, then Y represents the decimal number 3).

The 3 bits of Z represent a 3 bit binary number (e.g. if $Z = 101$, then Z represents the decimal number 5).

The output, Z, is to be given by the following:

$$Z = X - Y \quad \text{if } X > Y$$
$$Z = Y - X \quad \text{otherwise}$$

e.g. if $X = 101$ and $Y = 11$, then $Z = 010$. Likewise, if $X = 001$ and $Y = 11$, then $Z = 010$.

(a) Write a truth table that includes the output Z for all possible input combinations.

(a)

(b) Find a minimum SOP form for each of the three output bits.

X			Y				
x_1	x_2	x_3	y_1	y_2	z_1	z_2	z_3
0	0	0	0	0	0	0	0
0	0	0	0	1	0	0	1
0	0	0	1	0	0	1	0
0	0	0	1	1	0	1	1
0	0	ø	0	0	0	0	1
0	0	1	0	1	0	0	0
0	0	1	1	0	0	0	1
0	0	1	1	1	0	1	0
0	1	0	0	0	0	1	0
0	1	0	0	1	0	0	1
0	1	0	1	0	0	0	0
0	1	0	1	1	0	0	1
0	1	1	0	0	0	1	1
0	1	1	0	1	0	1	0
0	1	1	1	0	0	0	1
0	1	1	1	1	0	0	0

X			Y				
x_1	x_2	x_3	y_1	y_2	z_1	z_2	z_3
ø	0	0	0	0	1	0	0
1	0	0	0	1	0	1	1
1	0	0	1	0	0	1	0
1	0	0	1	1	0	0	1
1	0	1	0	0	1	0	1
1	0	1	0	1	1	0	0
1	0	1	1	0	0	1	ø
1	0	1	1	1	0	1	0
1	1	0	0	0	1	1	0
1	1	0	0	1	1	0	1
1	1	0	1	0	ø	0	0
1	1	0	1	1	0	1	1
1	1	1	0	0	1	1	1
1	1	1	0	1	1	1	0
1	1	1	1	0	1	0	1
1	1	1	1	1	1	0	0

(b) Z_1 is zero for all $X_1 = 0$

$y_1y_2 \backslash x_2x_3$	00	01	11	10
00	1	1	1	1
01	0	1	1	1
11	0	0	1	0
10	0	0	1	1

$$Z_1 = \longrightarrow (y_1' y_2' + x_3 y_1' + y_1' x_2 + x_2 x_3 + y_2' x_2)\, x_1'$$

Z_2 requires a five-variable Kmap

x_1 1/0

$y_1y_2 \backslash x_2x_3$				
	0/0	0/0	1/1	1/1
	0/0	0/0	1/1	0/0
	0/1	1/1	0/0	1/0
	1/1	1/0	0/0	0/0

Z_3 does as well

x_1 1/0

$y_1y_2 \backslash x_2x_3$				
	0/0	1/1	1/1	0/0
	1/1	0/0	0/0	1/1
	1/1	0/0	0/0	1/1
	0/0	1/1	1/1	0/0