KEY

[25 Points, a-g 3 points each, h 4 points] 1. Simplify each of the following expressions to minimum SOP form.

a. $f(a,b,c,d) = (b'+c'+d)(a'+b'+c')(a+b+c)(b+c+d)$

b. $f(a,b,c) = xy + x'yz' + yz$

c. $f(w,x,y,z) = w'x'y' + w'xz' + [(x+y+w'z)(x'+z'+wy')]'$

d. $f(w,x,y,z) = wxy' + (w'y' \equiv x) + (y \oplus wz)$. Recall a ⊕ b = a'b + ab', and (a ≡ b) = (a ⊕ b)'

e. $f(u,v,w,x,y,z) = (v'+u+w)(wx+y+uz') + (wx+uz'+y)$

f. $f(w,x,y,z) = (w+x')(y+z')(w+y)(x+y)(w+z)(x+z)$

g. Use QM to simplify 1b. Show your work for full credit.

h. $f(a,b,c,d) = a'bc'd + a'b'd + a'cd + abd + abc.$
Assume that ABCD = 0101, ABCD = 1001, ABCD = 1011 never occur.

(g) $xy + x'yz + yz =$

$xyz + xyz' + x'yz + xyz + x'yz$

111 110 010 111 011

(7) (6) (2) (3)

Column I

(2) 010
(3) 011
(6) 110
(7) 111

Column II

(2,3) 01–
(2,6) –10
(3,7) –11
(6,7) 11–

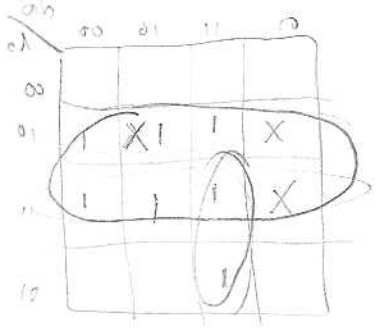
Column III

(2,3,6,7) –1–
(2,6,3,7) –1–

$\Rightarrow y$

xyz
xyz'
$x'yz$
xyz
$x'yz$

(h)



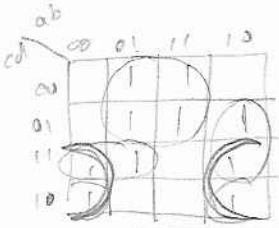
$\rightarrow \quad d + abc$

1

(a) $f = (b'+c'+d)(b'+c'+a')(b+c+a)(b+c+d)$

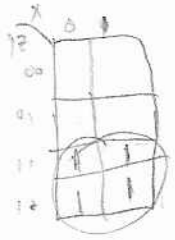
use $(x+y)(x+z) = x+yz$

$= (b'+c'+a'd)(b+c+ad) = b'b + b'c + b'ad + c'b + c'c + c'ad + a'db + a'dc + a'dad$



$= b'c + a'cd + ab'd + bc'$

(b)



→ y

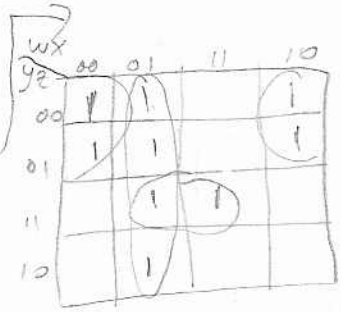
(c) $[(x+y+w'z)(x'+z'+wy')]' =$

$(x+y+w'z)' + (x'+z'+wy')' =$

$x'y'(w'z)' + xz(wy')' =$

$x'y'(w+z') + xz(w'+y') =$

$x'y'w + x'y'z' + xzw' + xzy$

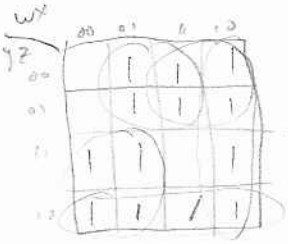


$= w'x + xyz + x'y'$

(d) note $a \equiv b = (a'b+ab')' = (a+b')(a'+b) = ab + a'b'$

$f = wxy' + (w'y'x + (w'y')'x') + y'wz + y(wz)'$

$= wxy' + w'y'x + (w+y)x' + y'wz + y(w'+z')$



$\Rightarrow yz' + wx' + w'y + xy'$

(e) $f = (wx + \ldots + uz')(v'+u+w+1) = wx + y + uz'$

(f) $(w+x')(y+z')(w+y)(x+y)(w+z)(x+z)$

$(w+y)(w+z) = w+yz$

$(x+y)(x+z) = x+yz$

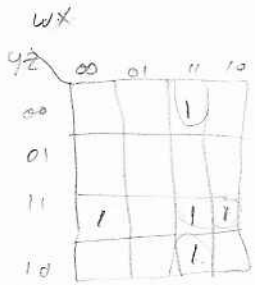
$= (w+x')(y+z')(w+yz)(x+yz)$

$= (w+x'yz)(y+z')(x+yz)$

$= (wy + wz' + x'yzy + x'yzz')(x+yz)$

$= (wy + wz' + x'yz)(x+yz)$

$= (wyx + wyyz + wz'x + wz'yz + x'yzx + x'yzyz)$



$= wyx + wyz + wxz' + x'yz$

$= \underline{wyz + wxz' + x'yz}$

[26 Points] 2. A combinational network has four inputs (A, B, C, and D), which represent a binary coded decimal digit. The network has two groups of outputs – S, T, U, V, and W, X, Y, Z. Each group represents a BCD digit. The output digits represent a decimal number which is 5 times the input number. For example, if ABCD = 0111, the outputs are 0011 0101. Assume that invalid BCD digits do not occur as inputs (this assumption ought to be reflected in the truth table).

[10] (a) Construct the truth table.

this is problem 5.27 from the homework

	A	B	C	D	S	T	U	V	W	X	Y	Z
0)	0	0	0	0	0	0	0	0	0	0	0	0
1)	0	0	0	1	0	0	0	1	0	1	0	1
2)	0	0	1	0	0	0	1	0	0	0	0	0
3)	0	0	1	1	0	0	1	1	0	1	0	1
4)	0	1	0	0	0	0	1	0	0	0	0	0
5)	0	1	0	1	0	0	1	0	0	1	0	1
6)	0	1	1	0	0	0	1	1	0	0	0	0
7)	0	1	1	1	0	0	1	1	0	1	0	1
8)	1	0	0	0	0	1	0	0	0	0	0	0
9)	1	0	0	1	0	1	0	0	0	1	0	1
	1	0	1	0	X	X	X	X	X	X	X	X
	1	0	1	1								
	1	1	0	0								
	1	1	0	1								
	1	1	1	0								
	1	1	1	1								
					X	X	X	X	X	X	X	X

don't care – invalid digits never occur

(b) From inspection:

$S = 0$ $\quad W = 0$

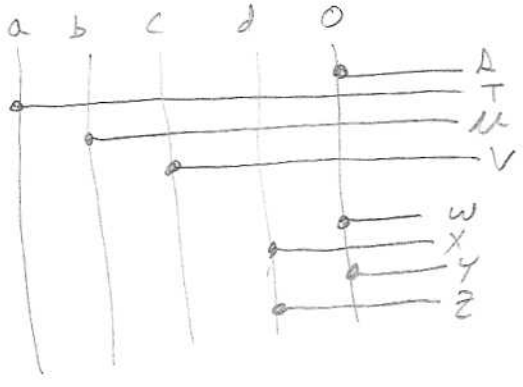
$T = A$ $\quad X = D$

$U = B$ $\quad Y = 0$

$V = C$ $\quad Z = D$

(can do Kmap instead)

(c)



[24 Points] 3. Complete the following conversion problems.

a. Write $wxy'+w'x'z+wy'z+w'yz'$ in minimal POS form.

b. Given $f'(a,b,c,d)=\prod M(0,1,2,3,7,9,15)$, find the minterm expansion for f (algebraic form)

c. Given $f(a,b,c,d)=\sum m(0,1,4,5,10,12)+\sum d(1,2,3,4)$, write f in minimum POS form.

d. Given $f(a,b,c,d)=\prod M(0,1,2,3,7,9,15)$, write f in minimum SOP form.

e. Draw $wxy'+w'x'z+wy'z'$ using only NAND gates.

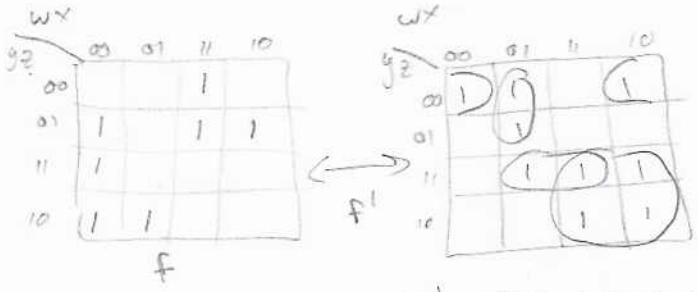
f. Draw $wxy'+w'x'z+wy'z'$ using only NOR gates.

g. Convert $f(a,b,c,d)=\sum m(0,1,4,5,10,12)+\sum d(1,2,3,4)$ to minimum SOP form.

h. Convert 1f into POS form.

(h)

3/(a)



$f' = wy + xyz + w'xy' + x'y'z'$

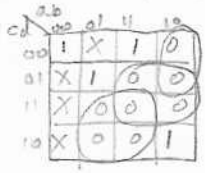
$f = (f')' =$

$(w'+y')(x'+y+z')(w+x'+y)(x+y+z)$

(b) $f' = \sum m(1,5,6,8,10,11,12,13,14)$ and $f = \sum m(0,2,3,7,9,15)$

$= a'b'c'd' + a'b'c'd + a'b'cd' + a'b'cd + a'bcd + ab'c'd + abcd.$

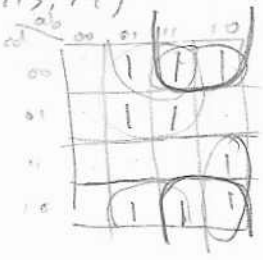
(c) $f = \sum m(0,5,10,12) + \sum d(1,2,3,4)$



$f' = bc + ad + ab'c'$

$f = (b'+c')(a'+d')(a'+b+c)$

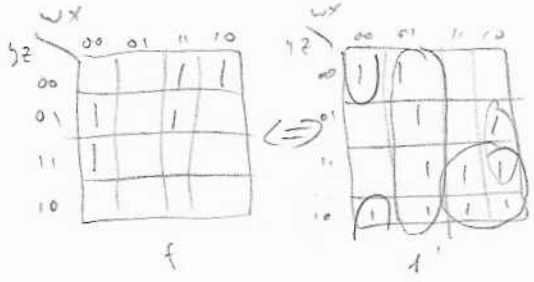
(d) $f = \sum m(4,5,6,8,10,11,12,13,14)$



$\rightarrow bc' + bd' + ad' + ab'c$

(e)

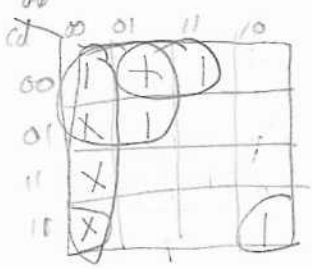
(f)



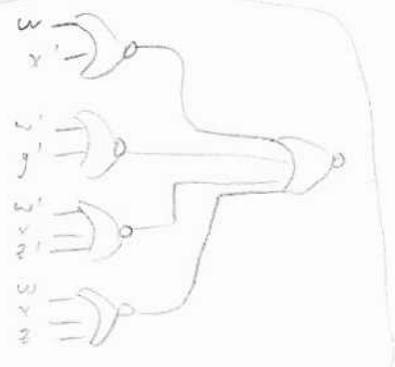
$f' = w'x + wy + wx'z + w'x'z'$

$f = (w+x')(w'+y')(w'+x+z')(w+x+z)$

(g) this is the same function as (c)



$f = b'c'd' + a'c' + bc'd'$



(h) already in POS form!

[25 Points] 4. A combinational network has four input bits A, B, C, and D. You are to design a network with an output Z (which consists of multiple bits). Z is a binary number whose value represents the number of 1's in the input. For example, if ABCD = 1001, the bits of Z represent the number 2.

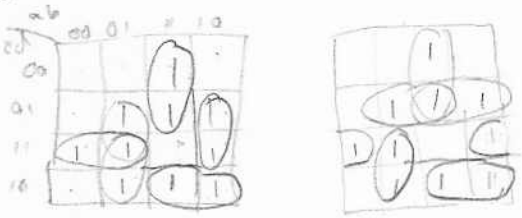
[2] (a) How many bits must Z have?

3 bits

[10] (b) Construct a truth table for the bits of Z.

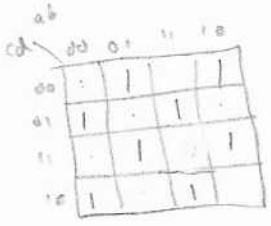
a b c d	z_1 z_2 z_3
0 0 0 0	0 0 0
0 0 0 1	0 0 1
0 0 1 0	0 0 1
0 0 1 1	0 1 0
0 1 0 0	0 0 1
0 1 0 1	0 1 0
0 1 1 0	0 1 0
0 1 1 1	0 1 1
1 0 0 0	0 0 1
1 0 0 1	0 1 0
1 0 1 0	0 1 0
1 0 1 1	0 1 1
1 1 0 0	0 1 0
1 1 0 1	0 1 1
1 1 1 0	0 1 1
1 1 1 1	1 0 0

(b) $z_1 = abcd$



need 6 terms:

$z_2 = abc' + ab'd + a'bd + a'bc + a'cd + acd'$



$z_3 = a'b'c'd + a'b'cd' + a'bc'd' + a'bcd + ab'c'd' + ab'cd + abc'd + abcd'$

(c)