Name:

Honor Code: KEY

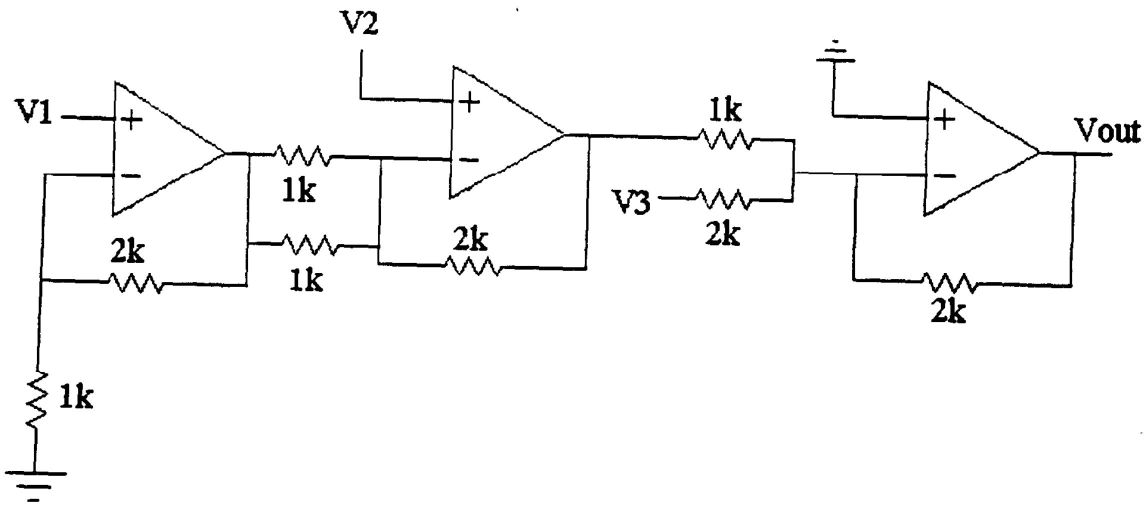
Instructions:

- Complete the 5 problems in the allotted time, and *report your answers in the box provided on this page.*
- Use the space on the accompanying pages to work the problems. Do not use a bluebook. Attach additional worksheets if necessary.
- If you wish to have partial credit awarded for any of your incorrect answers **you must write clearly and legibly.** Explain your work in words, if necessary.

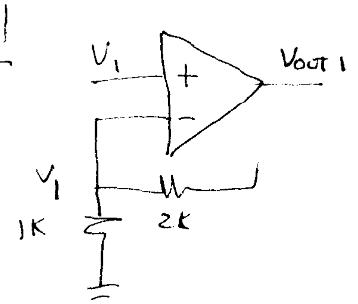
Good Luck.

Problem	**Answer**
1	Vout= $24V_1 - 10V_2 - V_3$
2a	Rth = 900 Ω
	Voc = .9 V
b	Rl = 900 Ω
c	MPT = .225 mW
3	12 V Ps: −7.6721 W
	6 A Cs: −60.1966 W
	-j2Ω Capacitor: 0 W always!
	2Ω Resistor: 17.1148 W
	4Ω Resistor: 15.3443 W
	6Ω Resistor: 35.4098 W
4	Vout = $15.1789\angle 18.4349^\circ$ Volts
5	R = 10 Ω L = 1 H

1. Find Vout in terms of V1, V2, and V3 assuming all opamps are ideal.



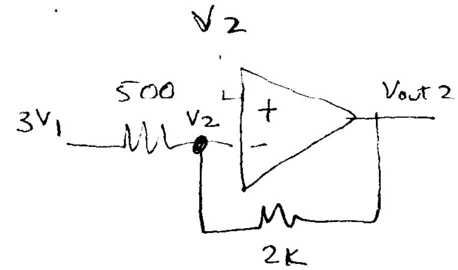
Opamp1



Nodal Analysis

$$\frac{V_1}{1K} + \frac{V_1 - V_{out1}}{2K} = 0 \qquad \underline{V_{out1} = 3V_1}$$

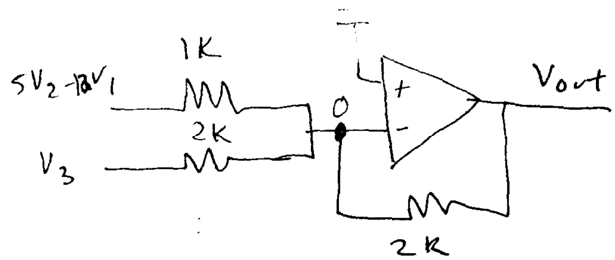
Opamp2



$$\frac{V_2 - 3V_1}{500} + \frac{V_2 - V_{out2}}{2000} = 0$$

$$\underline{V_{out2} = 5V_2 - 12V_1}$$

Opamp3



$$\frac{0 - (5V_2 - 12V_1)}{1000} + \frac{(0 - V_3)}{2000} + \frac{(0 - V_{out})}{2000} = 0$$

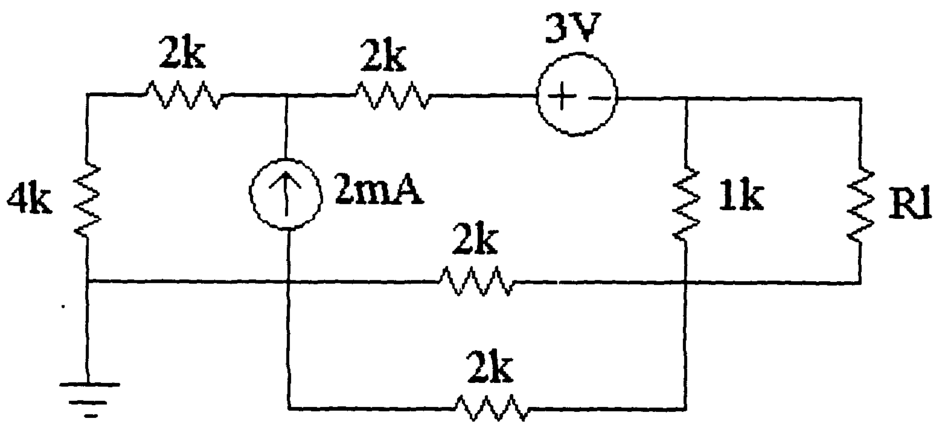
$$V_{out} = -2(5V_2 - 12V_1) - V_3$$

$$\boxed{V_{out} = 24V_1 - 10V_2 - V_3}$$

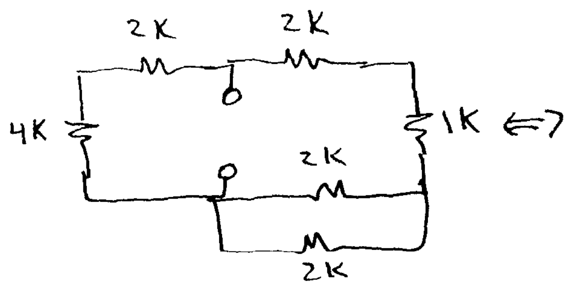
→ The Key to this problem is to break it up into 3 "sub-problems", each of which is easy to solve.

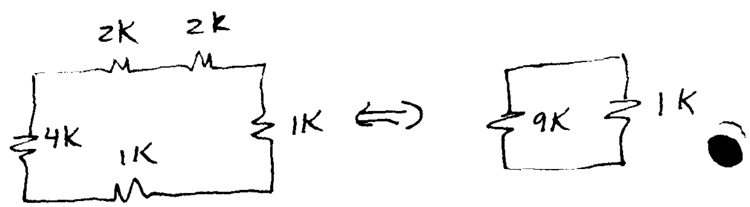
→ Also, you always want to write your nodal equations at the inputs of an opamp and not at the outputs. Why? because if you write nodal equations at the opamps output, you need to know how much current flows out of the opamp and you don't know that - it is not zero (in general).

2. Consider the circuit shown below.
   a) Find the Thevenin equivalent.
   b) Choose Rl for Maximum Power Transfer.
   c) Compute the Maximum Power Transferred.



a)

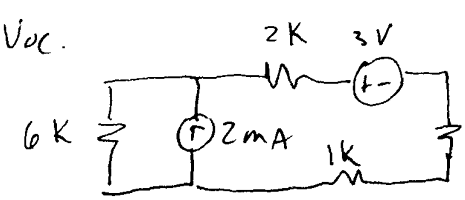


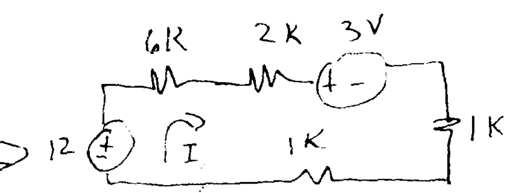


$R_{TH} = \frac{(9K)(1K)}{10K} = \underline{900\Omega}$

b) $R_L = R_{TH}$ for MPT $\quad R_L = 900\ \Omega$

a') Find V_{OC}.



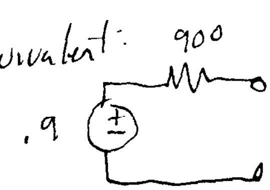


Source Transformation

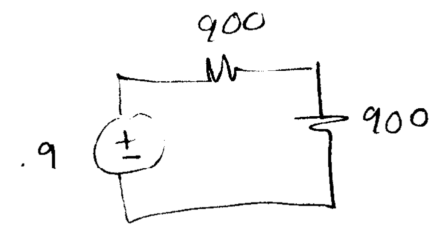
loop: $-12 + 10K + 3 = 0 \qquad I = .9mA$

$V_{OC} = (1000)(.9mA) = \underline{.9V}$

Thev. equivalent:



c) MPT



$V_L = .45$ Volts

$P_L = \frac{V_L^2}{R} = \frac{(.45)^2}{900} = \underline{.225\,mW}$

The key to this problem is to remember that when you find R_{TH}, you <u>cannot</u> simplify through the break-points. i.e. The 9K & 1K are in parallel, not in series.

also, finding V_{oc} is most easily done via source transformation; it is also reasonably straight-forward via loop analysis without source transformation.

Simplification techniques are very powerful!

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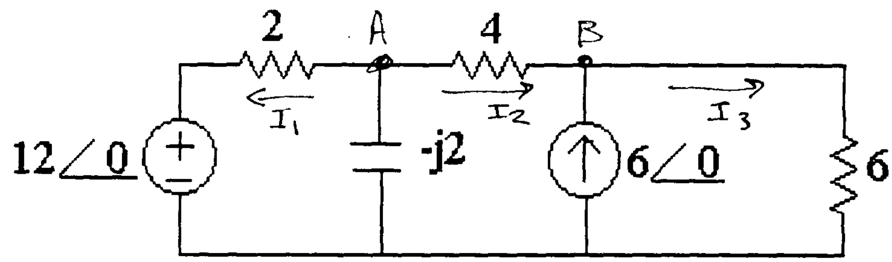
3. Calculate the average power absorbed/supplied by all the elements in this circuit



Nodal Analysis

$$\frac{A - 12\angle 0}{2} + \frac{A}{-j2} + \frac{A - B}{4} = 0 \qquad \frac{B - A}{4} - 6\angle 0 + \frac{B}{6} = 0$$

$$A\left(\frac{1}{2} + \frac{1}{-j2} + \frac{1}{4}\right) + B\left(-\frac{1}{4}\right) = 6\angle 0 \qquad B\left(5/12\right) + A\left(-\frac{1}{4}\right) = 6\angle 0 \quad (2)$$

mult by .6

$$(1) \quad A(.75 + .5j) + B\left(-\frac{1}{4}\right) = 6\angle 0 \qquad B(1/4) - .15A = 3.6\angle 0 \quad (2')$$

ADD

$$A(.75 + .5j - .15) = 9.6\angle 0$$

$$A = \frac{9.6\angle 0}{.6 + .5j} = 12.29154\angle -39.805^\circ \text{ Volts}$$

$B = (3.6\angle 0 + .15A)(4)$ From (2') above

$$= 14.4\angle 0 + .6A$$

$$= 14.4\angle 0 + .6(12.29154\angle -39.805^\circ)$$

$$= 20.0656 - j4.721$$

$$B = 20.6134\angle -13.2396^\circ \text{ Volts}$$

Find Currents:

$$I_1 = \frac{A - 12\angle 0}{2} = 4.1369 \angle -108.0038^\circ \text{ Amps}$$

$$I_2 = \frac{A - B}{4} = 2.7698 \angle -163.4941^\circ \text{ Amps}$$

$$I_3 = \frac{B}{6} = 3.4356 \angle -13.2396^\circ \text{ Amps}$$

Powers: $\frac{1}{2} I_m V_m \cos(\theta_V - \theta_I)$

12 VS: $\frac{1}{2}(4.1369)(12)\cos(+108.0038^\circ) = -7.6718$ W

6 AS: $-\frac{1}{2}(6)(20.6134)\cos(-13.2396^\circ) = -60.1965$ W

make sure to investigate absorb/supply status!

2Ω: $\frac{1}{2} I_M^2 R = \frac{1}{2}(4.1369)^2 2 = 17.1139$ W

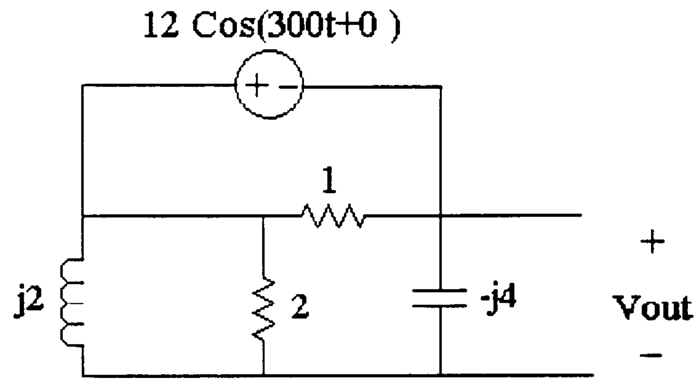
4Ω: $\frac{1}{2}(2.7698)^2 4 = 15.3436$ W

6Ω: $\frac{1}{2}(3.4356)^2 6 = 35.41$ W

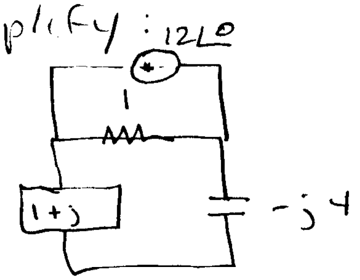
Note $\sum P \approx 0$.

this problem is not particularly difficult conceptually - just a little messy mathematically.

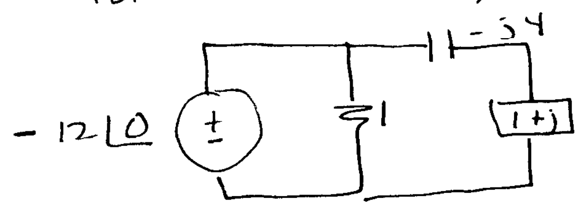
4. Find Vo in the following network.



First, simplify :



For convenience, redraw :



This is simply a voltage divider!

$$V_{cap} = -(12\angle 0)\left(\frac{-j4}{-j4+1+j}\right) = \frac{48j}{1-j3}$$

$$= -14.4 + 4.8j$$

$$= 15.1789 \angle 161.5651^{\circ} \text{ Volts}$$

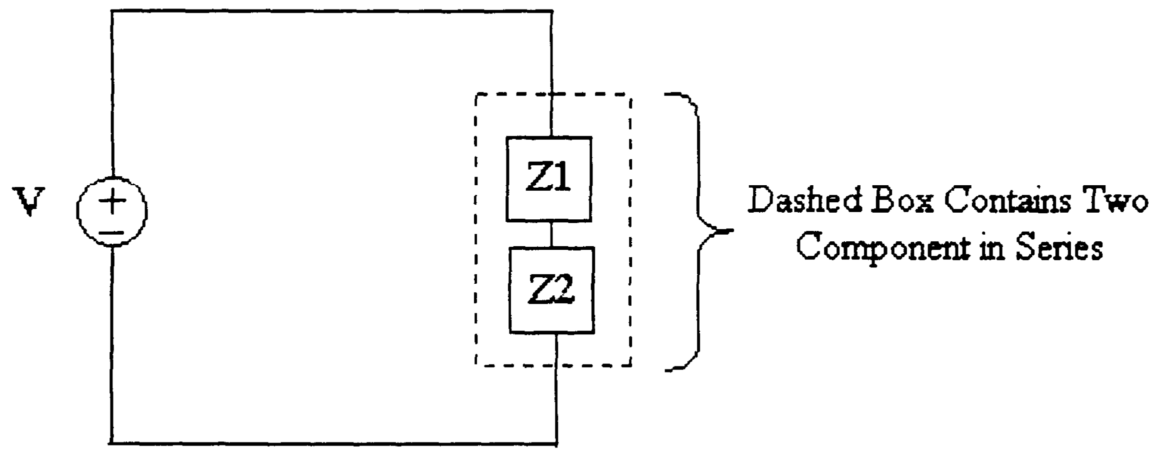
Notice a couple of things

1- Simplification turns this into an almost 'trivial' problem.

2- although it is drawn differently than we're used to, this is just a Voltage divider.

3- The $(-12\angle 0)$ requires a minus sign because the way the $+ -$ is drawn on the source.

5. You are given a box that is known to contain two components in series. (The components can be resistors, capacitors, inductors, or a combination of any two).



You make the following two measurements the total impedance of the box (Z):

a. Using an input sinusoid at 1 rad/sec as V, you find the phase of Z is 5.71°.
b. With an input sinusoid at 10 rad/sec as V, you find the magnitude of Z is $10\sqrt{2}$.

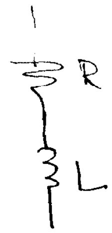
What are the components and what are their values? Given that there is no way to directly measure complex impedances in your lab, suggest a way that the measurements could have been made.

Step 1: What are the components?

a) tells us that the phase is 5.71° for $\omega = 1$. Since phase $= \tan^{-1}(\text{Imag}/\text{Real})$, we know that one component must be a resistor. (Since Real cannot be 0).

Furthermore, a phase of 5.71° indicates that imaginary part > 0, so the other component must be an inductor, since $Z_L = j\omega L$ and $Z_C = -j/\omega C$.

Now we know



$Z_1 = R$

$Z_2 = j\omega L$

or $Z_{EQ} = R + j\omega L$

Step 2: Values.

at $\omega = 1$, $Z_{EQ} = R + jL$, $5.71^\circ = \tan^{-1}(L/R)$

at $\omega = 10$ $Z_{EQ} = R + j10L$, $10\sqrt{2} = \sqrt{R^2 + 100L^2}$

Two equations and two unknowns:

$$\boxed{\begin{array}{l} L = 1\ H \\ R = 10\,\Omega \end{array}}$$

In lab, we'd set up V and measure I on the scope. Using $\underline{V} = \underline{I}\,\underline{Z}$, we can then calculate $\underline{Z}$ for various ω.

If we didn't have a scope that allowed measurement of $\underline{I}$ (we don't), we could use the RMS meter to measure magnitudes and proceed as before.