Multiple Model Nonlinear Filtering for Low Signal Ground Target Applications

Chris Kreucher*, Keith Kastella

Veridian Systems, Ann Arbor, MI

ABSTRACT

This paper describes the design and implementation of multiple model nonlinear filters (MMNLF) for ground target tracking using Ground Moving Target Indicator (GMTI) radar measurements. The MMNLF is based on a general theory of hybrid continuous-discrete dynamics. The motion model state is discrete and its stochastic dynamics are a continuous-time Markov chain. For each motion model, the continuum dynamics are a continuous-state Markov process described here by appropriate Fokker-Plank equations. This is illustrated here by a specific two-model MMNLF in which one motion model incorporates terrain, road, and vehicle motion constraints derived from battlefield observations. The second model is slow diffusion in speed and heading. The target state conditional probability density is discretized on a moving grid and recursively updated with sensor measurements via Bayes' formula. The conditional density is time updated between sensor measurements using Alternating Direction Implicit (ADI) finite difference methods. In simulation testing against low signal to clutter + noise Ratio (SNCR) targets, the MMNLF is able to maintain track in situations where single model filters based on the either of the component models fail. Potential applications of this work include detection and tracking of foliage-obscured moving targets.

Keywords: Nonlinear filtering, Fokker-Planck equation, target tracking, multiple models, alternating direction implicit scheme, finite difference methods

1. INTRODUCTION

Ground target tracking often requires operation at low signal to clutter + noise ratios (SNCR), especially when target vehicles are moving beneath a forest canopy or otherwise obscured. However, ground targets typically have constraints on their motion that constitute an additional information source that can be included in target motion models. For example, terrain, roads and other surface features may have been previously mapped. This additional information source, when properly taken advantage of, can help to overcome the low SNCR problem. Past work [4,5] shows that tracking performance is improved by incorporating this type information into filters based on variable structure interacting multiple model Kalman Filters (IMMKF). In [4,5] the IMMKF have spatially varying non-isotropic plant noise but do not directly incorporate dynamic vehicle inputs such as preferred heading or speed. In [1], a single model nonlinear filter using an Inhomogenous Integrated Ornstein-Uhlenbeck (IIOU) model with spatially varying coefficients is used. This nonlinear filter performs satisfactorily in geographic regions that have been well surveyed. In these regions, the model coefficients are well characterized. However, performance is poor in areas where preferred heading and speed are unknown.

The principal contribution of the work reported here is the development of an interacting multiple model nonlinear filter (MMNLF), where one model incorporates vehicle motion preferences directly into the filter. In wartime or other realistic situations, this data may not be reliable for all areas of interest. Some areas will have incomplete or missing data, either due to lack of vehicle histories or conflicting vehicle histories. It is therefore necessary to allow the filter to operate without these control inputs in these situations. In this work, this is addressed by the second model, which is a constant speed/constant heading model. Based on sensor measurements, the MMNLF adaptively weights the models and uses the one most appropriate for the current region.

This paper is organized as follows. Section 2 first synopsizes NLF and then extends the framework to multiple models. Section 3 gives the details of our implementation of a two-model NLF. An IIOU model is used for target motion model that incorporates preferred vehicle motion derived from terrain, road and vehicle dynamics constraints. The second model is a simple constant velocity (CV) model. Together, these two models give the benefit of using *a priori* terrain information and a

* Correspondence: Email: Christopher.Kreucher@veridian.com; Telephone (734) 994-1200 x 2717; Fax: (734) 994-5905; Mail: Veridian, P.O. Box 134008, Ann Arbor, MI 48113-4008.

procedure for areas where that information is unavailable. Simulations comparing the multiple model filter with each of the single model filters are presented in Section 4. The simulations show that the multiple model filter is able to perform well in situations where each of the two filters acting alone has difficulty.

2. NONLINEAR FILTERING MATHEMATICAL FRAMEWORK

The structure of a nonlinear filter (NLF) is similar to that of a conventional Kalman filter in that it consists of time update (or prediction) and measurement update. Unlike the Kalman Filter, which estimates just the first two moments of the conditional density, nonlinear filters develop an estimate of the entire target state conditional density. This difference allows NLF based trackers to operate robustly in many real-life situations where one or more of the linear/Gaussian assumptions of the Kalman filter are violated.

2.1 Single Model NLF

The objective of a filter is to estimate the time-dependent target state $\mathbf{x}_t$, given a sequence of measurements $\mathbf{y}_k$ made at discrete times t_k. We denote by Y_t the collection of measurements $\mathbf{y}_k$ up to and including time t_k: $Y_t = \{\mathbf{y}_\mathbf{t} : t_k \le t\}$. To treat nonlinear effects, NLF methods maintain an estimate of the conditional density $p(\mathbf{x}_t \mid Y_t)$ for the target state $\mathbf{x}_t$ conditioned on the collection of measurements Y_t. Quantities of interest such as the minimum mean square error state estimate, the covariance, and the maximum likelihood state estimate can be constructed from this density.

The starting point for modeling time evolution (prediction) in the NLF is the same as it is in Kalman filtering: the Ito stochastic differential equation. The Ito equation summarizes how target states and their probability densities evolve in time due to both deterministic and random target motion effects. In the case of target tracking, this is often called the target motion model or kinematic model, because it is often constrained by the physical properties of the target being tracked. For the time-dependent target state $\mathbf{x}_t$, the Ito equation is given by

$$d\mathbf{x}_t = \mathbf{f}(\mathbf{x}_t, t)dt + \mathbf{G}(\mathbf{x}_t, t)d\beta_t, \quad t \ge t_0 \tag{1}$$

where $\mathbf{x}_t$ and $\mathbf{f}$ are n-vectors, $\mathbf{G}$ is an $n \times r$ matrix function and $\{\beta_t, t \ge t_0\}$ is an r-vector Brownian motion process with $E\{d\beta_t d\beta_t^{\mathrm{T}}\} = \mathbf{Q}(t)dt$. In the Ito equation, $\mathbf{f}(\mathbf{x}_t, t)dt$ characterizes the deterministic portion of the state transition from time *k* to time *k+1*, while $\mathbf{G}(\mathbf{x}_t, t)d\beta_t$ characterizes the random portion. Note that both $\mathbf{f}$ and $\mathbf{g}$ can be nonlinear.

Because NLF tracks the entire probability density function (PDF), an equation for the time evolution of the density must be derived using the Ito equation. It can be shown that this is given by the Fokker-Planck equation (FPE)

$$\frac{\partial p}{\partial t}(\mathbf{x}_t \mid Y_{t_k}) = L\big(p(\mathbf{x}_t \mid Y_{t_k})\big), \quad t_k \le t < t_{k+1} \tag{2}$$

where

$$L(p) \equiv -\sum_{i=1}^{n} \frac{\partial(\mathbf{f}_i p)}{\partial \mathbf{x}_i} + \frac{1}{2}\sum_{i,j=1}^{n} \frac{\partial^2\big(p(\mathbf{G}\mathbf{Q}\mathbf{G}^{\mathrm{T}})_{ij}\big)}{\partial \mathbf{x}_i \partial \mathbf{x}_j} \tag{3}$$

with initial condition given by $p(\mathbf{x}, t_k \mid Y_{t_k})$. This prediction equation is solved efficiently on a grid using a Dyakonov [3] finite difference scheme (an alternating direction implicit scheme) to generate the predicted density $p(\mathbf{x}_{t_k} \mid Y_{t_{k-1}})$.

Discrete-time measurements are incorporated into the NLF through the likelihood $p(\mathbf{y} \mid \mathbf{x})$, which is based on a physical model for the probability to obtain measurement $\mathbf{y}$ given the target state is $\mathbf{x}$. In the case of target tracking, this is often called the sensor model, because it models the way a sensor makes measurements of the entity being tracked. When the measurements are conditionally independent and depend only on the instantaneous state of the target, a measurement $\mathbf{y}_{t_k}$ is used to generate $p(\mathbf{x}_{t_k} \mid Y_{t_k})$ from the predicted density $p(\mathbf{x}_{t_k} \mid Y_{t_{k-1}})$ using Bayes' formula

$$p(\mathbf{x}_{t_k} \mid Y_{t_k}) = \frac{p(\mathbf{y}_{t_k} \mid \mathbf{x}_{t_k}) p(\mathbf{x}_{t_k} \mid Y_{t_{k-1}})}{\int d\mathbf{x}'_{t_k} \, p(\mathbf{y}_k \mid \mathbf{x}'_{t_k}) p(\mathbf{x}'_{t_k} \mid Y_{t_{k-1}})} \tag{4}$$

In this expression, $\mathbf{y}_{t_k}$ denotes the actual measurement vector obtained at time t_k while $\mathbf{x}_{t_k}$ is a variable. This time and measurement updated density can be used to generate the minimum mean square error target state estimate $\hat{\mathbf{x}}_{t_k}$ at time t_k

$$\hat{\mathbf{x}}_{t_k} = \int d\mathbf{x}_{t_k} \mathbf{x}_{t_k} \, p(\mathbf{x}_{t_k} \mid Y_{t_k}) \tag{5}$$

with covariance

$$\mathbf{P}_t = \int d\mathbf{x}_{t_k} \left(\hat{\mathbf{x}}_{t_k} - \mathbf{x}_{t_k}\right)\left(\hat{\mathbf{x}}_{t_k} - \mathbf{x}_{t_k}\right)^{\mathrm{T}} p(\mathbf{x}_{t_k} \mid Y_{t_k}) \tag{6}$$

The specific implementation of the NLF, then, relies on the development of two entities: (a) The Ito equation (or Fokker-Plank equation) to model the way that the state changes from time k to time $k+1$ (the target motion model), and (b) the measurement model likelihood, which models the way observations are made (the sensor model).

2.2 Multiple Model NLF Development

The Interacting Multiple Model Nonlinear Filter (MMNLF) allows for adaptive model switching in systems whose behavior changes over time. The multiple model system obeys one of r models, all of which are characterized by their own target state conditional probability density function $p_m(\mathbf{x}_t \mid Y_t)$ where $m = 1 \ldots r$. In the two-model example developed below, we will have $m = \{\text{IIOU}, \text{CV}\}$. The target dynamics for each of the r models are described by their own (different) Ito stochastic differential equation

$$d\mathbf{x}_t = \mathbf{f}_m(\mathbf{x}_t, t)dt + \mathbf{G}_m(\mathbf{x}_t, t)d\beta_t, \quad t \geq t_0 \tag{7}$$

Although not strictly necessary, we assume for simplicity that all models have the same n-dimensional state vector $\mathbf{x}_t$.

Each of the conditional probability density functions $p_m(\mathbf{x}_t \mid Y_t)$ is constructed recursively according to Bayes' formula as before:

$$p_m(\mathbf{x}_{t_k} \mid Y_{t_k}) = \frac{p_m(\mathbf{y}_{t_k} \mid \mathbf{x}_{t_k}) p_m(\mathbf{x}_{t_k} \mid Y_{t_{k-1}})}{\int d\mathbf{x}'_{t_k} \sum_{m'} p_{m'}(\mathbf{y}_{t_k} \mid \mathbf{x}'_{t_k}) p_{m'}(\mathbf{x}'_{t_k} \mid Y_{t_{k-1}})} \tag{8}$$

Where the sensor model $p_m(\mathbf{y}_{t_k} \mid \mathbf{x}_{t_k})$ may be unique for different models. The MMNLF is implemented by keeping all filters active at all times. At each time step, a mixing of the probabilities from each of the models to each of the others is undertaken. This allows soft mode switching and for the eventual accrual of (measurement-based) evidence to dictate which motion model is being used at a particular time.

Defining the probability of transition from model i to model j by Λ_{ij}, we rewrite the FPE to incorporate this mode mixing as

$$\frac{\partial p_m}{\partial t}(\mathbf{x}_t \mid Y_{t_k}) = L_m\left(p_m(\mathbf{x}_t \mid Y_{t_k})\right) + \sum_{m'} \Lambda_{m',m} p_{m'}(\mathbf{x}_t \mid Y_{t_k}), \quad t_k \leq t < t_{k+1} \tag{9}$$

The new FPE includes the usual Ito equation terms as well as a mixing of probabilities from the different models. Note that the total probability must be conserved. Therefore, if probability enters model m from model m', model m' loses some of its the probability when it is updated (i.e. $\Lambda_{m'm'} < 1$).

From this, $p_m(\mathbf{x}_t \mid Y_{t_k})$ gives the probability that the target is in state $\mathbf{x}_t$ and model *m*. Furthermore, $\sum_m p_m(\mathbf{x}_t \mid Y_{t_k})$ gives the probability that the target is in state $\mathbf{x}_t$ and any model, and $\sum_{\mathbf{x}} p_m(\mathbf{x}_t \mid Y_{t_k})$ gives the probability that the target is obeying model m. The minimum mean square error target state estimate $\hat{\mathbf{x}}_{t_k}$ at time t_k is modified to incorporate the distributions in all models as

$$\hat{\mathbf{x}}_{t_k} = \int d\mathbf{x}_{t_k} \mathbf{x}_{t_k} \sum_m p_m(\mathbf{x}_{t_k} \mid Y_{t_k}) \tag{10}$$

with covariance modified similarly

$$P_t = \int d\mathbf{x}_{t_k} \left(\hat{\mathbf{x}}_{t_k} - \mathbf{x}_{t_k}\right)\left(\hat{\mathbf{x}}_{t_k} - \mathbf{x}_{t_k}\right)^{\mathrm{T}} \sum_m p_m(\mathbf{x}_{t_k} \mid Y_{t_k}) \tag{11}$$

The MMNLF implementation, then, requires prediction and measurement models for each of the *r* filters used. Furthermore, a model mixing matrix Λ_{ij} which models how the target moves from one model to another must be developed.

3. NONLINEAR FILTERING IMPLEMENTATION

Here, we consider the case of tracking a ground target in a low signal application. The target state is modeled using the 4-dimensional state

$$\mathbf{x} = (x, y, \theta, v)^{\mathrm{T}} \tag{12}$$

where *x* and *y* are the target's Cartesian location in the topocentric plane (meters) centered on the region of interest, θ is the target heading and v is the target speed (m/s). We use a 2-model MMNLF, where one model incorporates preferred vehicle motion preferences and the second does not. The first model is useful in areas where the vehicle motion preferences are well characterized. The second model is useful where the vehicle motion preferences are either missing or poorly characterized.

3.1 The IIOU Ground Target Motion Model

We assume first that the area has been previously surveyed and spatially varying vehicle motion preferences have been developed (as in [1], for example). In the IIOU-based target motion model, we want to incorporate these vehicle motion preferences into the filter. Ito equations that couple the target dynamics to the spatially varying vehicle motion preferences can be constructed using Inhomogenous Integrated Ornstein-Uhlenbeck (IIOU) models. Given a spatially varying preferred speed $v_0(x,y)$, and a mean time to speed corrections $\tau_v(x,y)$, the Ito equation is

$$dv = -\frac{1}{\tau_v(x,y)}\left(v - v_0(x,y)\right)dt + d\beta_v \tag{13}$$

where $d\beta_v$ is a white Brownian motion process with power spectral density with expected value $E\left(d\beta_v{}^2\right) = q_v(x,y)dt$ where

$$q_v(x,y) = \frac{2}{\tau_v(x,y)}\sigma_v^2 \tag{14}$$

and σ_v^2 is the variance of the speed deviation from its preferred value.

Similarly, given a spatially varying preferred heading $\theta(x,y)$ and a mean time to heading corrections $\tau_\theta(x,y)$, the Ito equation is

$$d\theta = -\frac{1}{\tau_\theta(x,y)}\left(\theta - \theta_0(x,y,\theta)\right)dt + d\beta_\theta \tag{15}$$

The IIOU model includes a 180-degree ambiguity in θ, as targets will typically align themselves parallel or anti-parallel to the preferred heading θ. Specifically, the adjusted preferred heading $\theta_0(x,y,\theta)$ is given by

$$\theta_0(x,y,\theta) = \begin{cases} \varphi(x,y), & |\theta - \varphi(x,y)| < \pi/2 \\ \varphi(x,y) + \pi, & \text{otherwise,} \end{cases} \tag{16}$$

where $\varphi(x,y)$ is the preferred direction. The power spectral density of the heading process is $E\left(d\beta_\theta{}^2\right) = q_\theta(x,y)dt$ where $q_\theta(x,y) = \dfrac{2}{\tau_\theta(x,y)}\sigma_\theta^2$.

Of course, we also write $dx = v\cos(\theta)$ and $dy = v\sin(\theta)$. The full set of Ito equations for this model are then

$$dx = v\cos(\theta) \tag{17}$$

$$dy = v\sin(\theta)$$

$$d\theta = -\frac{1}{\tau_\theta(x,y)}\left(\theta - \theta_0(x,y,\theta)\right)dt + d\beta_\theta$$

$$dv = -\frac{1}{\tau_v(x,y)}\left(v - v_0(x,y)\right)dt + d\beta_v$$

Defining $\dot{x} = v\cos(\theta)$ and $\dot{y} = v\sin(\theta)$, the FPE for this model (hereafter called the IIOU model) becomes

$$L_{\text{IIOU}}(p) = -\dot{x}\frac{\partial p}{\partial x} - \dot{y}\frac{\partial p}{\partial y} + \frac{1}{\tau_\theta(x,y)}\frac{\partial}{\partial\theta}\left(\left(\theta - \theta_0(x,y,\theta)\right)p\right) + \frac{1}{\tau_v(x,y)}\frac{\partial}{\partial v}\left(\left(v - v_0(x,y)\right)p\right) + \tag{18}$$

$$\frac{1}{2}q_\theta(x,y)\frac{\partial^2 p}{\partial\theta^2} + \frac{1}{2}q_v(x,y)\frac{\partial^2 p}{\partial v^2}$$

3.2 Constant Speed Motion Model

There are several instances in which using vehicle motion preferences in the NLF is not desirable. First, there are times or locations when previous observations are not available. More often, there are locations where vehicle tracks conflict, indicating that there are either no paths through an area or multiple paths. In these instances, a constant speed/constant heading (CV) model is preferred.

In this case, we use the Ito equations

$$dx = v\cos(\theta) \tag{19}$$

$$dy = v\sin(\theta)$$

$$d\theta = d\beta_\theta$$

$$dv = d\beta_v$$

Defining $\dot{x} = v\cos(\theta)$ and $\dot{y} = v\sin(\theta)$ as before, the FPE for this model (which we refer to hereafter as the CV model) is

$$L_{CV}(p) = -\dot{x}\frac{\partial p}{\partial x} - \dot{y}\frac{\partial p}{\partial y} + \frac{1}{2}q_\theta(x,y)\frac{\partial^2 p}{\partial \theta^2} + \frac{1}{2}q_v(x,y)\frac{\partial^2 p}{\partial v^2} \tag{20}$$

3.3 Measurement Model

In both the CV and IIOU models, we model the target measurements as square-law detected return amplitude on a uniform grid of size $M = N_x \times N_y$ (for simplicity, we ignore the usual Doppler estimate obtained as part of the GMTI measurement). The amplitude in pixel i for scan k is $y_{k,i}$ and the entire scan is $\mathbf{y}_k = \{y_{k,i} \mid i = 1,\ldots,M\}$. Let $i_\mathbf{x}$ denote the target-containing pixel. For a target with SNCR λ (here assumed known), the probability distribution for the amplitude in pixel $i_\mathbf{x}$ is

$$p_1(y_{i_\mathbf{x}}) = \frac{1}{1+\lambda}\exp\left(-y_{i_\mathbf{x}}/(1+\lambda)\right) \tag{21}$$

The distribution in the empty cells is $p_0(y_{i_\mathbf{x}}) = \exp\left(-y_{i_\mathbf{x}}\right)$. The density for the entire scan is

$$p(\mathbf{y}_k \mid \mathbf{x}_k) = \kappa\, p_1(y_{k,i_\mathbf{x}}) / p_0(y_{k,i_\mathbf{x}}) \tag{22}$$

where κ is a target-state independent constant that can be discarded in the Bayes' formula update.

3.4 Model Mixing

In our application, we expect that once entering a model m (either the IIOU or the CV model), the target will continue to obey that model with high probability. Furthermore, transition probabilities, while small, will tend to favor the target entering the IIOU model. We use

$$\Lambda = \begin{bmatrix} -.1 & .2 \\ .1 & -.2 \end{bmatrix} \tag{23}$$

4. SIMULATION RESULTS

The MMNLF described above was implemented and a generalized form of ADI was used to solve for the multiple model probability on a regular moving grid with Bayes' formula (Eq. (8)) used for measurement update. To evaluate the MMNLF performance, we used the hospitability for maneuver (HM) map shown in Figure 1. HM [6,7,8] is a spatially varying quantity characterizing the ease with which a vehicle can traverse a particular area, with higher values being more hospitable for travel. The spatial resolution of HM is quite fine, typically on the same order as the Digital Terrain Elevation Data (DTED). Here we assume that targets prefer to follow regions of high HM. This corresponds to following the hospitability ridges, and moving with a speed that is proportional to the hospitability. For a more detailed analysis, as well as alternative methods of developing preferred heading and speed, see [1].

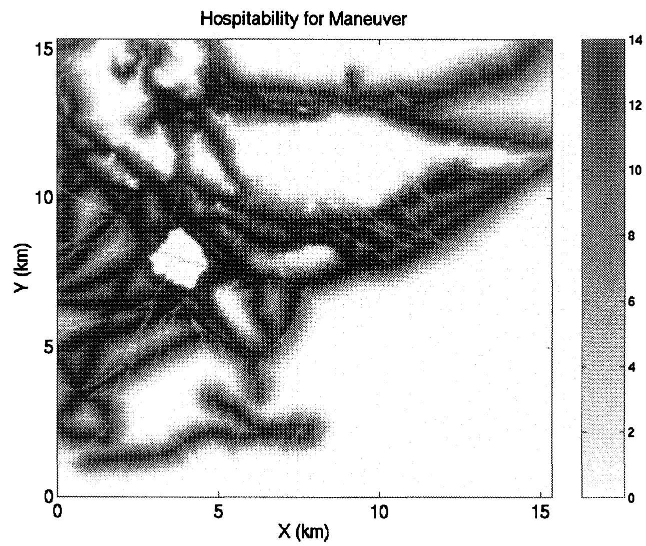

Figure 1. An example HM map.

To illustrate the potential utility of MMNLF, we perform two sets of simulations and compare the MMNLF tracker performance with two single model NLF's: one using just the CV model, and one using just the IIOU model. In both sets simulations, the parameters of the filters are fixed at $\sigma_v = .1$ m/s and $\sigma_\theta = .3°$ (recall that $\sigma_i^2 = \tau_i q_i / 2$, Eq. (14)). The simulated vehicle trajectories are generated using the Ito equations from the IIOU model. In the first set of simulations, there is no model mismatch between the heading and speed standard deviations and the filter parameters, while in the second set of simulations, the standard deviations are much larger than the IIOU parameters.

In the first simulation, the power spectral densities of the simulated trajectory are identical to those IIOU filter models and the SNCR is set at a relatively low value of 6 dB. Since the same model is used in both the simulated trajectory and the filter, we expect that the IIOU model will perform well in tracking this target. On the other hand, the CV model should perform poorly, since the low SNCR makes the measurements unreliable. Indeed, as is shown in Figure 2 and Figure 3, the NLF with the IIOU based model is best able to track the target. The CV model tracks well until the target makes a complicated maneuver at t = 180, at which time it loses track. The MM based filter also tracks the target very well, and is able to handle the maneuver that derailed the CV model.

Notice further that both the IIOU and MM, which are aided by the underlying vehicle motion preferences, provide more accurate speed and heading estimates.

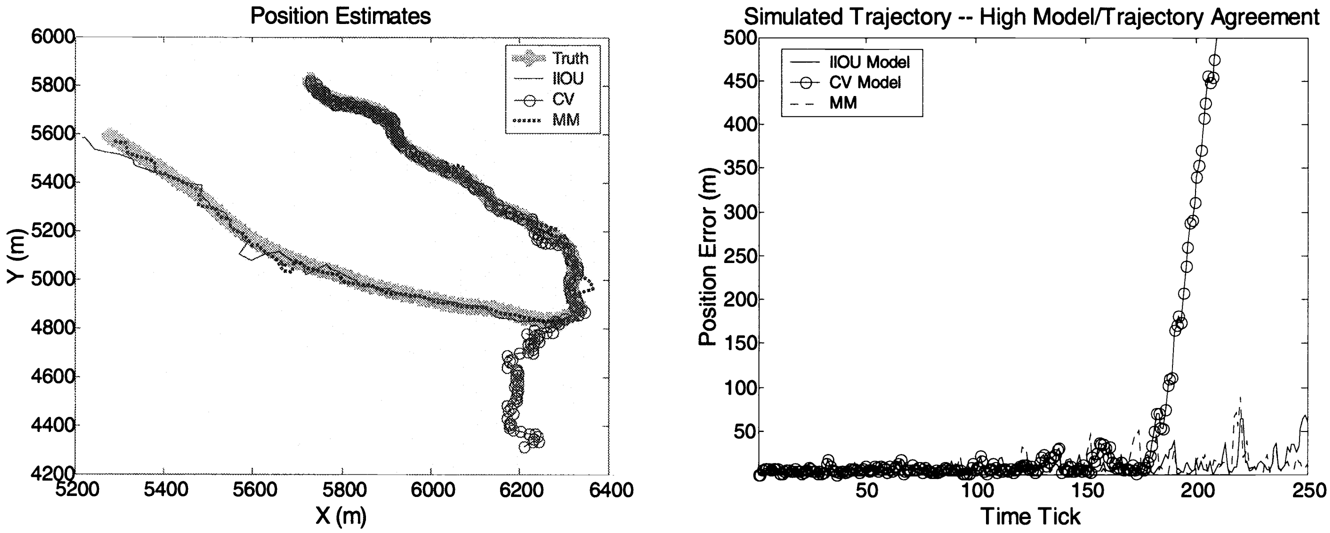

Figure 2. (L) The target trajectory and the different trackers position estimates. Both the IIOU and MM trackers hold the target, while the CV model loses it. (R) The position RMSE for each time step.

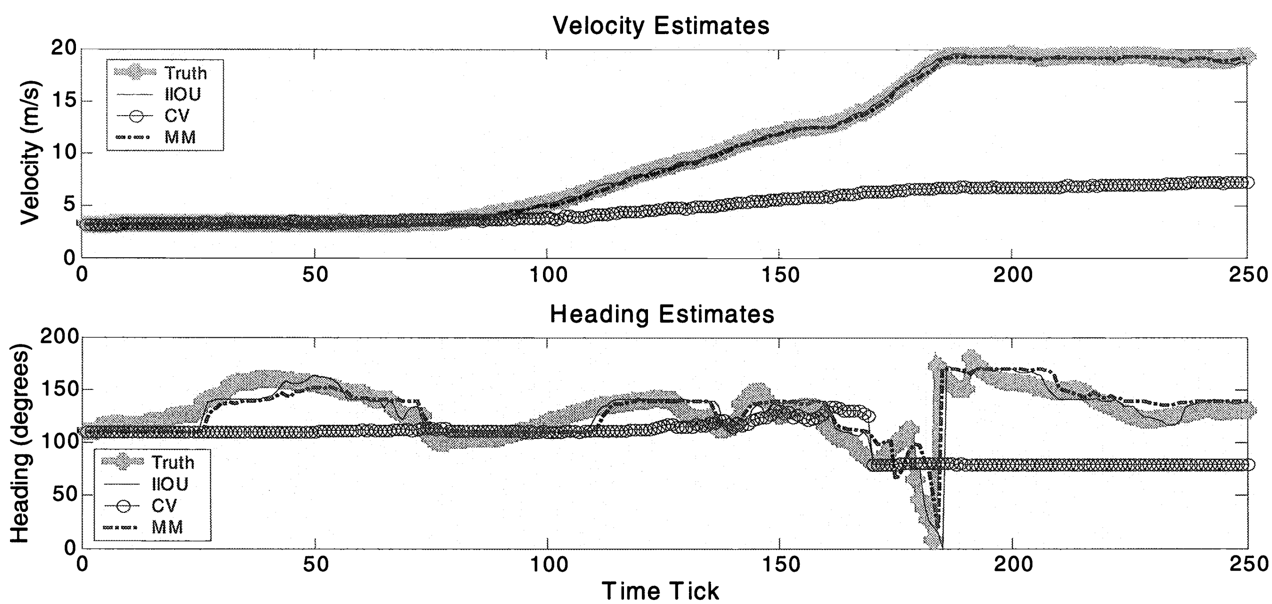

Figure 3. (T) The speed estimates of the three trackers show that the CV model is unable to hold on to the target as it speeds up. (B) The heading estimates show that the complicated maneuver at t=180 causes the CV to lose track.

Table 1 shows the root mean squared and root median squared errors for the three trackers. Since the CV model loses the target, the position error becomes unbounded.

	Pos. (m)	Heading (deg.)	Vel. (m/s)
IIOU	14.94	17.76	0.16
CV	453.53	39.84	7.29
MM	15.85	16.50	0.19

	Pos. (m)	Heading (deg.)	Vel. (m/s)
IIOU	6.45	7.43	0.11
CV	9.31	26.43	3.98
MM	6.74	9.98	0.13

Table 1. (L) The root-mean squared error for the three trackers. (R) The root-median squared error.

Figure 4 shows the model probabilities in the MMNLF. Note that most of the time, the IIOU probability is near .7, with a few brief excursions away from that value. The reason for this is that most of the time, both models have some validity. Therefore, during typical operation the model probabilities will settle in at a spot dictated by the mixing matrix. However, when one model clearly fits the target motion better than the other, that model will take over. Figure 4 shows that near t=175 (when the measurements become inconsistent with the CV model and the CV filter loses the target), most of the probability of flows into the IIOU model of the MM filter. After that short-term event, both models become viable again and the probability mix returns to what it was before.

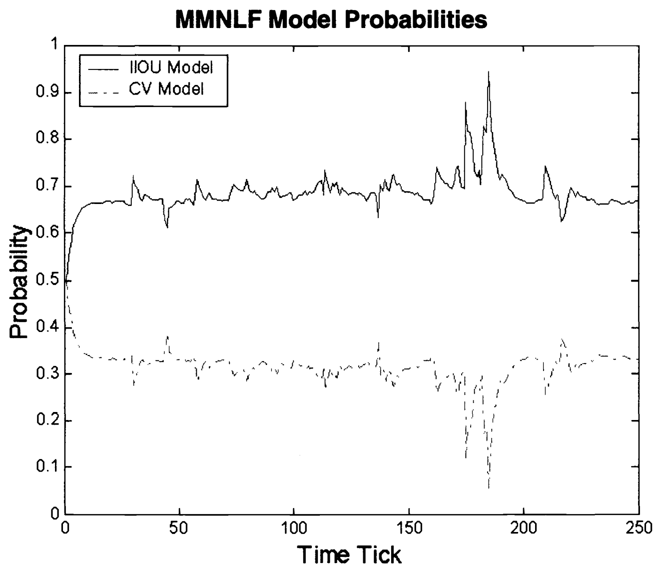

Figure 4. MMNLF Model Probabilities for Simulation 1.

In the second set of simulations, the variances of the simulated trajectory are increased substantially from that of the filter to $\sigma_v = 1$ m/s and $\sigma_\theta = 1.5°$. We furthermore increase the SNCR to 10dB, which means that the measurements are more reliable than in the first simulation. In this case, we expect that the IIOU model will have difficulty tracking the target because the preferred speed and heading it imposes on the target are severely violated. We furthermore expect the CV model, which does not impose these values on the target, to perform well.

Figure 5 shows the position estimates of each of three trackers. The CV and MM filters track the target very well throughout. The IIOU filter has trouble at around t=40 due to a maneuver that is contrary to the underlying vehicle motion preferences. Notice in Figure 6 that the MM filter (which uses the IIOU most of the time) switches into the CV model between t=43 and t=50 and thus avoids the high position errors incurred at that time. Figure 7 shows the speed and heading as tracked by the three filters. Table 2 gives the root-mean and root-median squared errors.

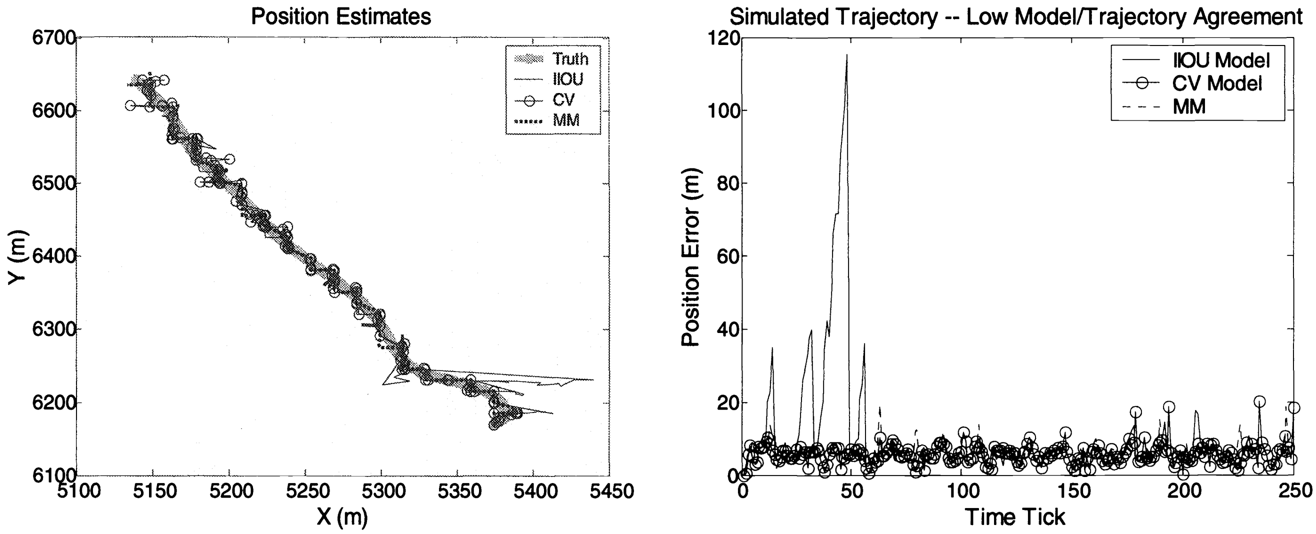

Figure 5. (L) The target trajectory and the different trackers position estimates. (R) The position RMSE for each time step.

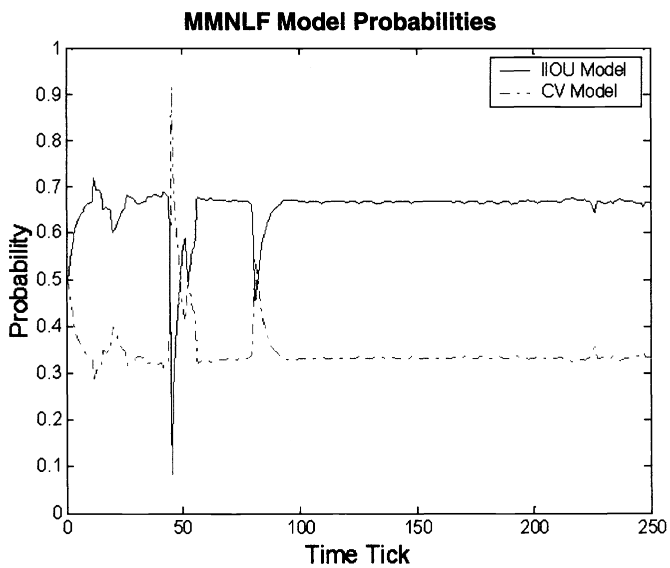

Figure 6. The model probabilities show that the filter mostly acts in the IIOU mode until the targets behavior warrants a switch to the CV mode at around t=40.

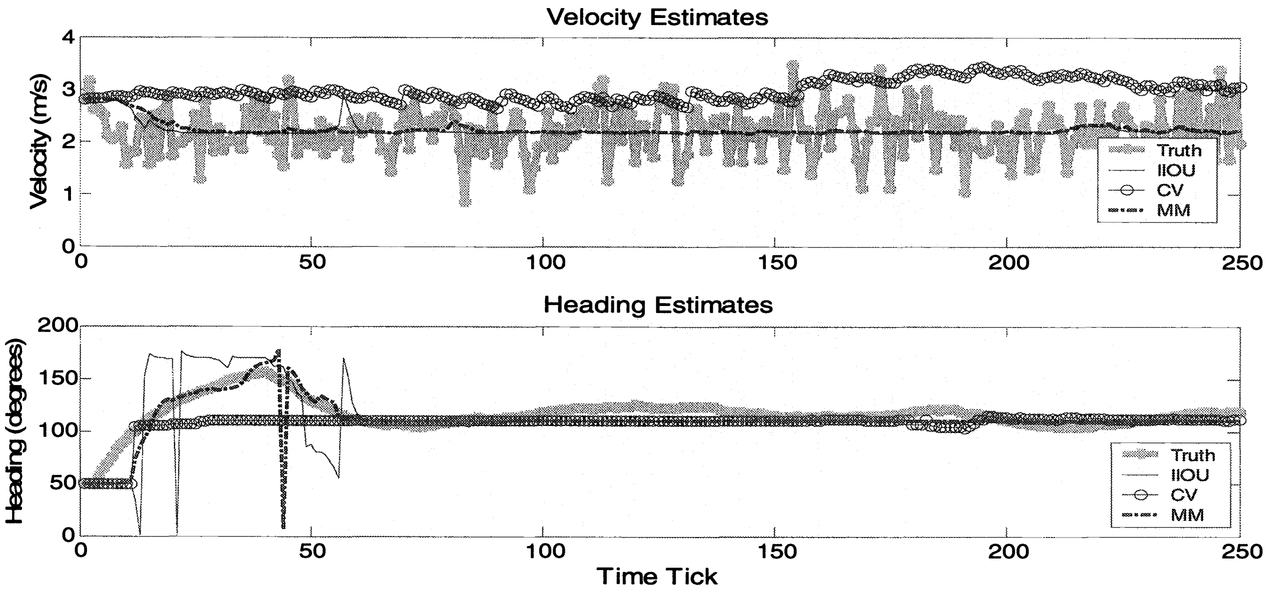

Figure 7. The speed and heading estimates of the three filters.

	Pos. (m)	Heading (deg.)	Vel. (m/s)
IIOU	18.13	20.52	0.49
CV	6.48	15.39	0.94
MM	6.61	12.95	0.48

	Pos. (m)	Heading (deg.)	Vel. (m/s)
IIOU	6.23	6.98	0.33
CV	5.84	6.96	0.78
MM	5.92	5.49	0.33

Table 2. (L) The root-mean squared error for the three trackers. (R) The root-median squared error.

5. CONCLUSION

This paper has described the design and implementation of multiple model nonlinear filters for ground target tracking. In this implementation, the target state conditional density is discretized on a moving grid, and the estimated target location is a given by the minimum mean square state estimate. Target motion is described using Fokker-Plank equations, and measurements are incorporated via Bayes' formula. We have demonstrated one specific two-model filter, which includes a purely diffusive motion model and a motion model based on previous observations. In the simulations presented here, the multiple model method performs well in situations where either filter acting on it's own has difficulty.

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