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**Multi-Sensor  
Fusion**

# Graph-Entropic Approaches for Multi-Sensor Image Registration

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# Agenda

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- Outline of Graph-Entropic Registration Approach
- Model-based georeferencing with a DEM reference
- Graph matching and the tie-point matching problem
- Examples & Applications



# Two key elements of our approach

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- Use of digital elevation model (DEM) as reference
  - **Significant reduction** in registration parameter search space:  
  
{All perspective/projective transforms & associated perturbations}  
-> {collection geometry parameters}
- Novel graph-entropic cost function
  - **Generalizes** pixel-to-pixel mutual-information (MI) image registration to more conventional tie-point based registration
  - **Statistically robust** procedures to allow for missing/extra tie-points due to mismatch between multi-sensor phenomenologies
  - **Reduced computational complexity** relative to previous MI based approaches



# Multi-Sensor Registration Approach

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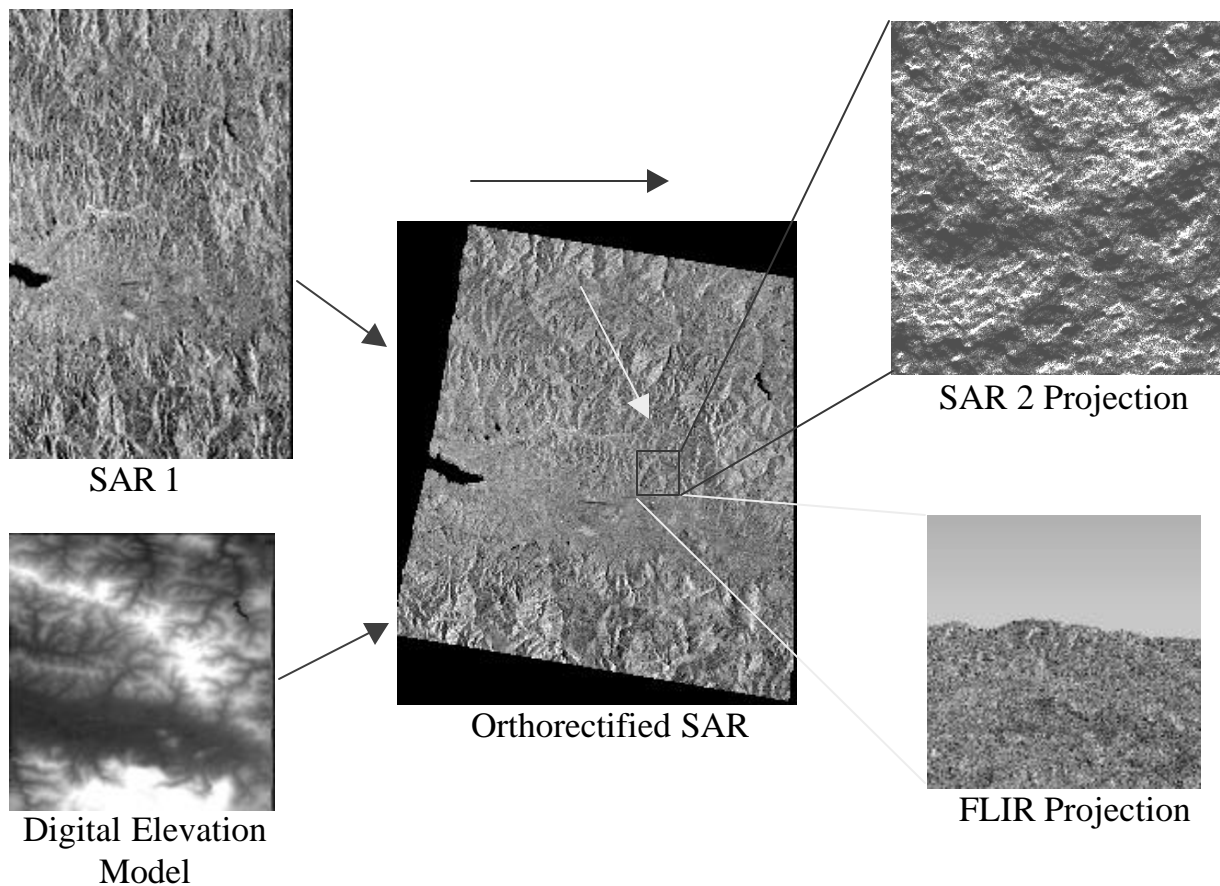
- Nominally a three-step process for SAR  $\leftrightarrow$  HSI/MSI image registration:
  1. **SAR  $\leftrightarrow$  DEM:** Georeference SAR imagery via DEM-based registration
  2. **HSI/MSI  $\leftrightarrow$  DEM:** Georeference EO imagery to DEM ref.
  3. **SAR  $\leftrightarrow$  HSI/MSI:** Reproject both image sets onto common viewing plane; further refine registration by matching common tie-points
- Each step requires solution of a similar image matching problem



# Example: Multi-Sensor Registration Approach

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# Tie-point Based Image Registration

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- Assume the existence of tie-points, common spatial landmarks:
  - $\{Y_1, Y_2, \dots, Y_M\}$  from the test image
  - and  $\{X_1, X_2, \dots, X_N\}$  from the reference image
- Tie-points can be selected via various means:
  - Natural terrain features such as shadows & discontinuities caused by terrain relief, features from man-made objects such as roads, buildings, agriculture, etc.
- Registration Problem: estimate and invert the mapping  
 $T: X \rightarrow Y$ 
  - For flat terrain: degenerates to {projective, rotation, scale} transformations



# Graph-Entropic Tie-point registration

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- Basic Setup:  $\{X_j\}_{j=1}^M$   $\{Y_k\}_{k=1}^N$  are spatial point processes in  $\mathbb{R}^d$  representing tie points from a pair of images to be registered
- Assume *a priori* unknown distributions:  $X_j \sim f$   $Y_k \sim g$
- Also assume there exists a (possibly non-differentiable) mapping  $T : X \rightarrow Y$  that relates the tie points



# Graph-Entropic Tie-point registration

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- Define:  $D_n(f \parallel g) = \frac{1}{1-n} \ln \int_{R^d} f^n(x) g^{1-n}(x) dx$ 
  - $v=1$ : Kullback-Leibler number (pseudo metric)
  - $v= 1/2$ : Hellinger distance (symmetric, true metric)
- Minimization problem:  $T^* = \arg \min_{T \in \Psi} D_n(f \parallel g)$ 
  - $D_n$  is Renyi "divergence" of order  $v$  between  $f$  and  $g$
  - $\Psi$  is set of all mappings consistent with 3D->2D projections of DEM reference

*Registration parameters  $T^*$  obtained via minimizing  $D_n$*





# Computational Strategies

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- **Indirect**, via density estimation: estimate  $f, g$  and integrate  
-> computationally intensive, problematic for large-dimensions (i.e., vector images), density estimates can be unstable
- **Direct**, via minimum spanning tree (MST): [Beardwood, Halton, Hammersly]  
-> ***Renyi entropy  $R_n$   $\leftrightarrow$  weighted length  $L_n$  of MST***



# Computational Strategies

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- By BHH theorem,  $L_n$  converges to  $R_n$  for large enough  $n$

- Also, the joint Renyi entropy of  $X, Y$  satisfies:

$$R_v(X, Y) \geq R_v(X, T^{-1}X) \geq R_v(X)$$

- Therefore, can equivalently select  $T$  to minimize the Renyi entropy of the joint tie-point set,  $\{X, T^{-1}X\}$
- Equivalently (via BHH theorem): choose the  $T$  for which we find the smallest length minimum spanning tree



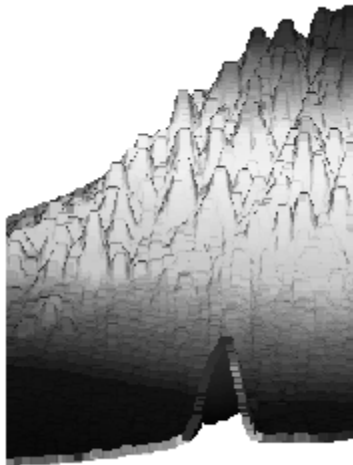
# Synthetic Data

## Example: SAR <-> EO

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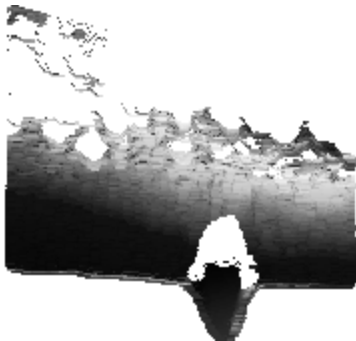
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- Projections of a DEM into EO & SAR viewing planes



EO projection formed at  $\{300,0,90\}$  with solar illumination at  $\{300,0,90\}$

Colormap indicates terrain height



SAR projection taken from  $\{300,0,90\}$



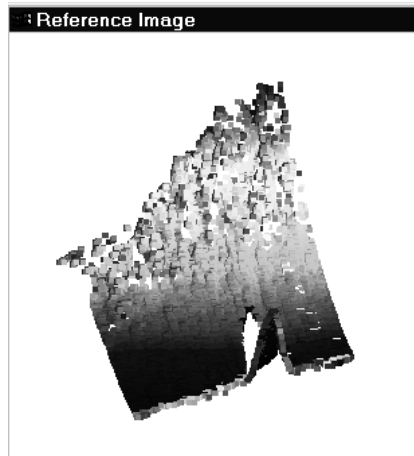
# Synthetic Data

## Example: SAR <-> EO

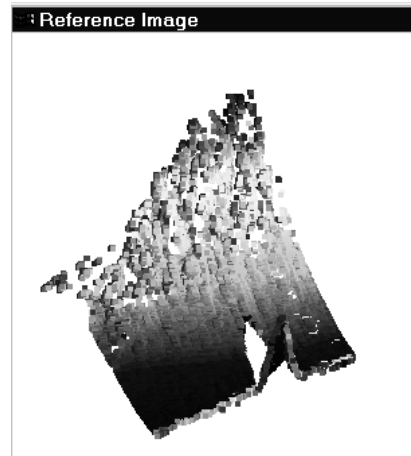
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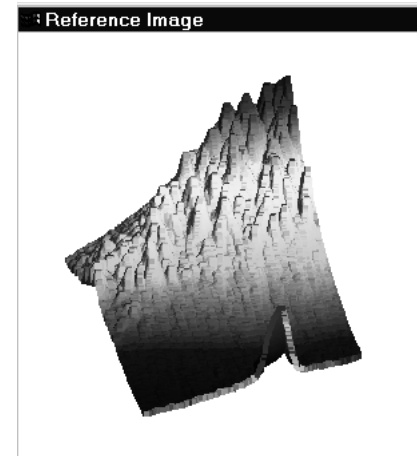
- Matching a reprojected SAR view to an EO view [colormap indicates height]
  - Gaps show missing information in SAR -> EO reprojection due to shadowing



SAR plane reprojected to correct EO view  
(300,0,110)



SAR plane reprojected to an incorrect EO view  
(300,0,120)



reference image:  
EO projection at  
(300,0,110)

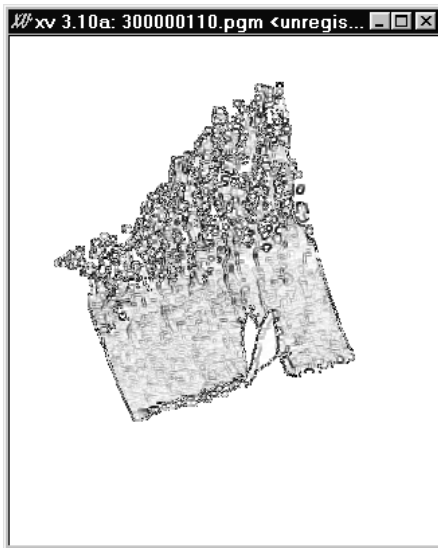


# Minimum-Divergence Matching of Edges

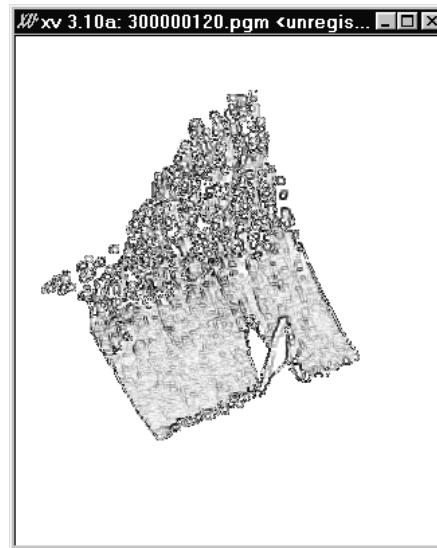
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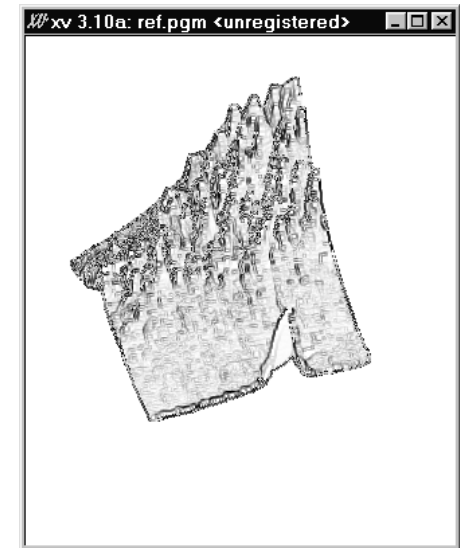
- Edges as tie-points: edge maps extracted from intensity images (via Marr-Hildreth operator)



SAR plane reprojected  
to correct EO view  
(300,0,110)



SAR plane reprojected  
to an incorrect EO view  
(300,0,120)

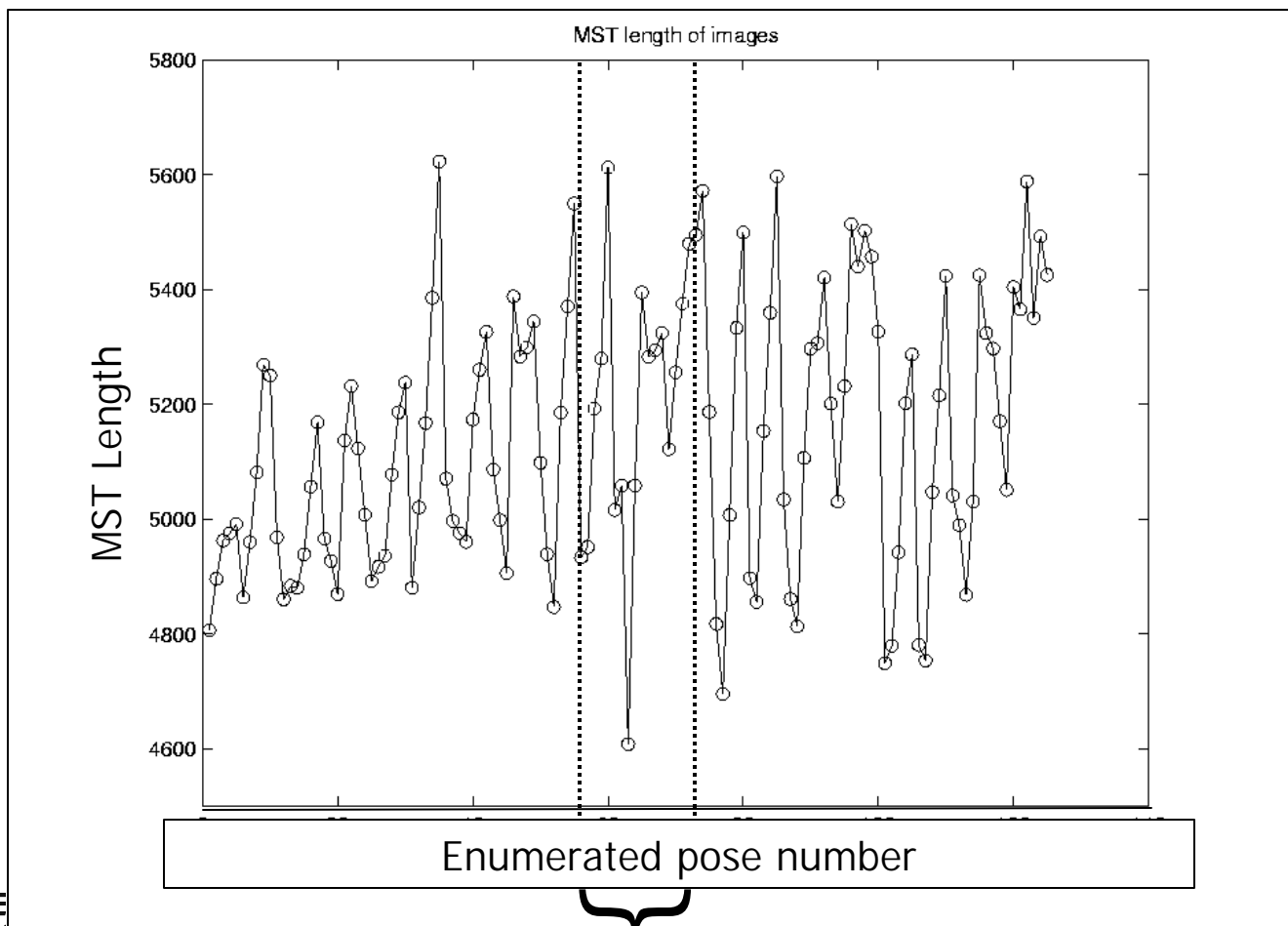


reference image:  
EO projection at  
(300,0,110)

# MST length for various poses (i.e. various T)

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# Project Status

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- Graph-entropic image registration approach currently being developed on Delivery Order under AFRL STIL MSF program
- Current capabilities:
  - Portable C/OpenGL based code for rendering arbitrary SAR, EO/IR views (Solaris, Linux, WinNT)
  - Automatic tie-point extraction from edges or local entropy
  - Correlation-based matching
  - Soon to arrive: Graph-entropy based matching



# Summary

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- Novel approach for tie-point based image registration
- Generalizes notion of pixel-based mutual information registration to mutual information on graphs induced by tie-points
- Ongoing research into robustness: impact of mismatch due to sensor differences in phenomenology, viewing geometries, illumination conditions
  - Approach: model  $f$  and  $g$  as contaminated mixtures w/ unknown contaminating distributions





# Backup Charts...

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- How does this compare to MI approach of Viola et. al.?
  - > We're looking at the entropy of the graph induced by the tiepoints
  - > This should be compared to their pair-wise pixel-pixel MI approach



# Mutual Information Metrics

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- $i_1$  and  $i_2$  -- two images to be registered
- Assume  $i_2(\mathbf{y}) = F[i_1(T(\mathbf{x})), \mathbf{p}]$ ,
  - $T(\mathbf{x})$  represents the unknown geometric distortion between the images, and
  - $F[.]$  is a possibly unknown function that maps pixel intensities in image  $i_1$  into intensities in  $i_2$ .
- One approach: parameterize  $T(\mathbf{x})$  according to sensor collection geometry, illumination models, say  $T_f(\mathbf{x})$
- Select  $\phi^*$  to maximize the mutual information between  $i_1$  and  $i_2$  :  $\phi^* = \operatorname{argmax}_{\phi} I(i_1, i_2(T_f^{-1}(\mathbf{y})))$

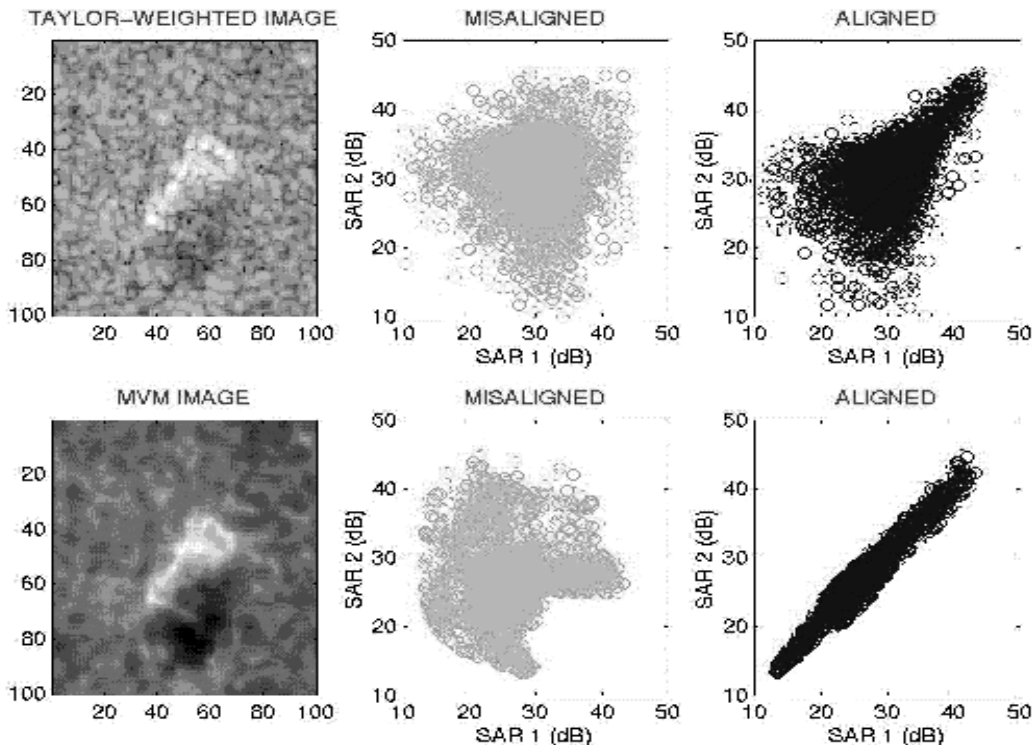


# Image Alignment via Maximization of MI

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- Pixel-pixel SAR-SAR matching:
  - Plot of single-pixel intensities in misaligned test image vs. pixel intensities in ref. image.



## Heuristic:

Mutual information  $\leftrightarrow$  spread of the plot

Smaller spread  $\rightarrow$  Larger MI

## Observation:

SAR speckle noise destroys the image-image correlation and efficacy of pixel-pixel MI

Alternative: look for "stable" match features

